Consistent Downscaling of Seismic Inversions to Cornerpoint Flow Models
SPE 103268

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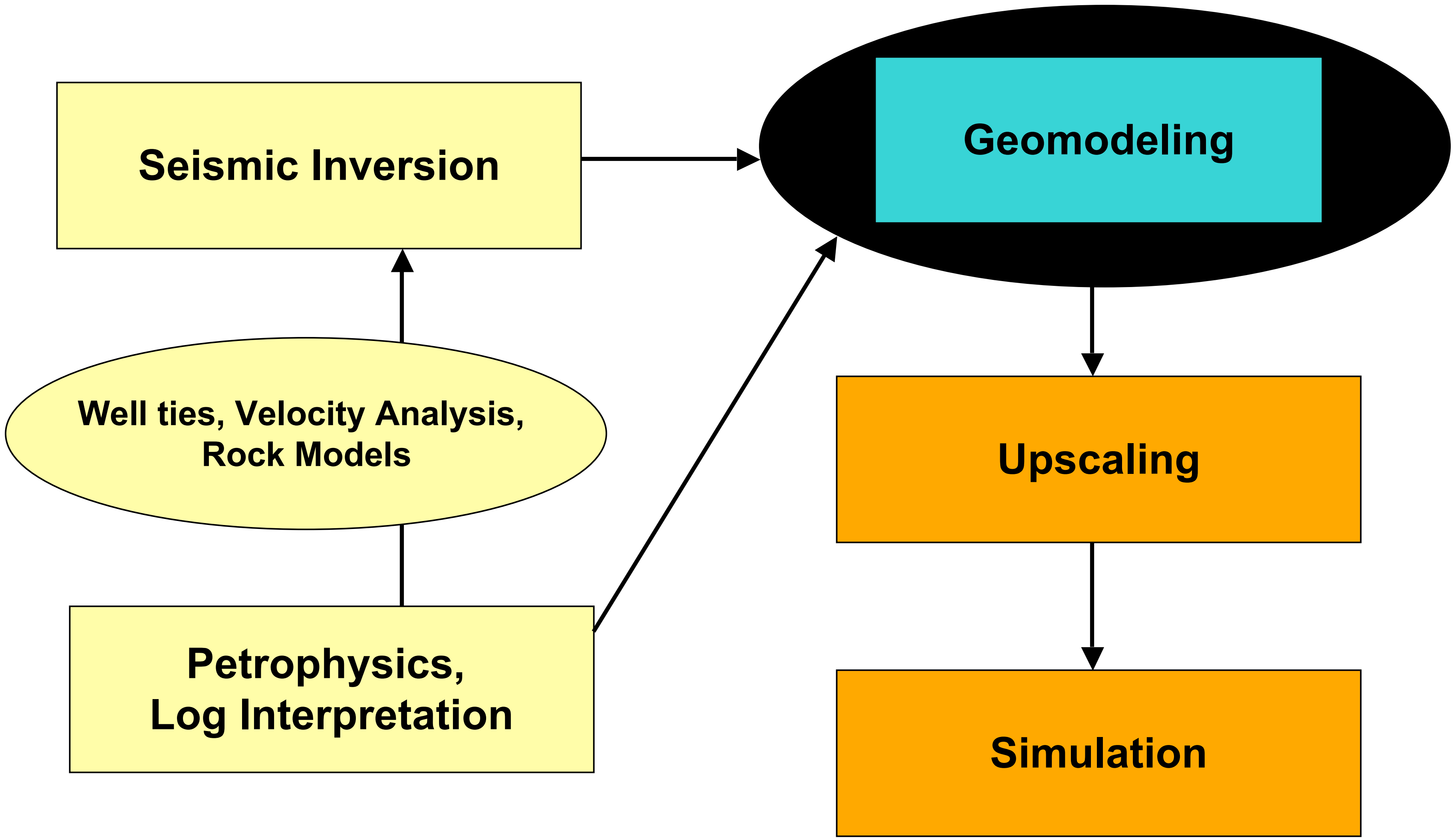
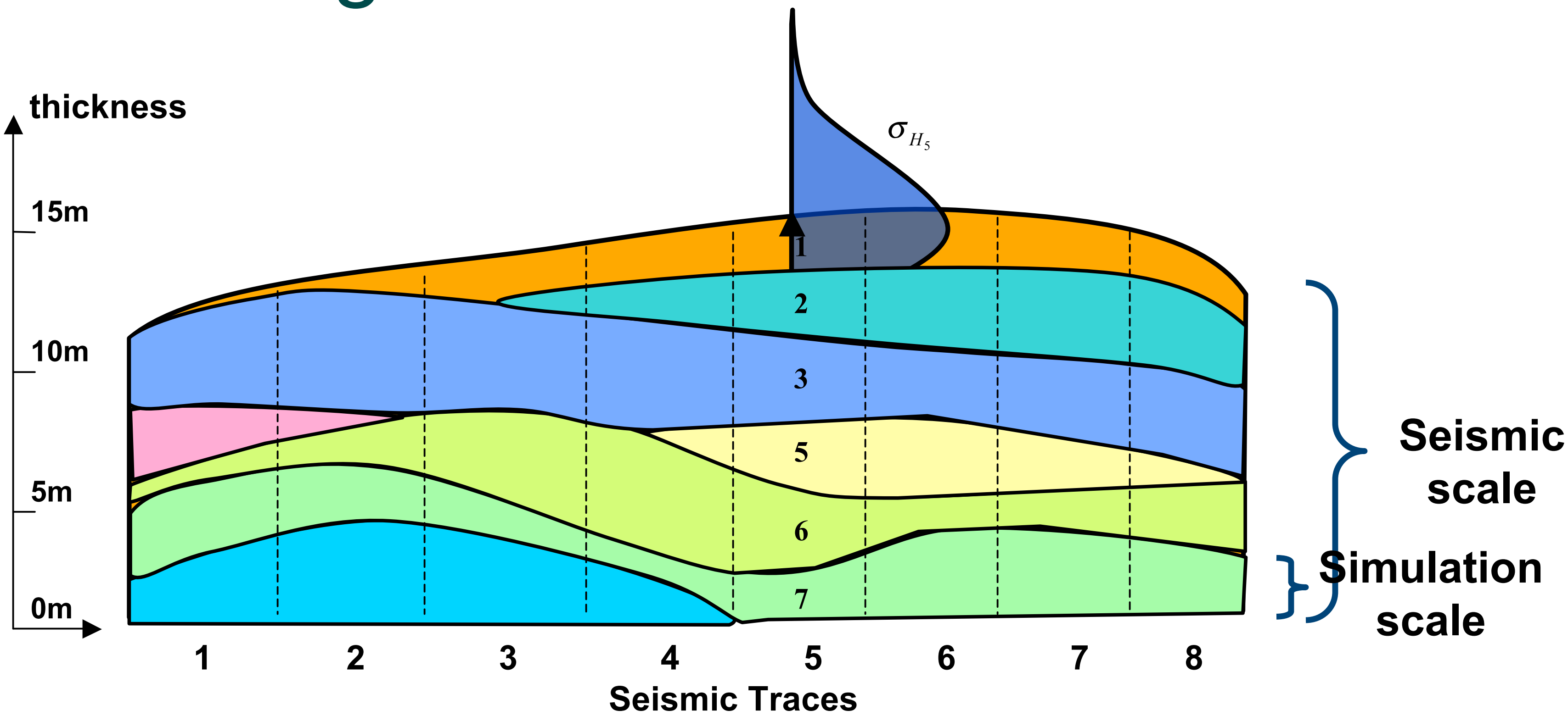
Overview



Gunning et al 2006

Integration of multi scale data



Sum of layer thicknesses simulated should approximately match seismic thickness

Methods

Bayesian Inference

Posterior — **Likelihood** — **Prior**

$$P\left(\mathbf{t}\middle|H,\mathbf{d}_{\ell k}\right)=\frac{P\left(H\middle|\mathbf{t},\mathbf{d}_{\ell k}\right)P\left(\mathbf{t}\middle|\mathbf{d}_{\ell k}\right)}{P\left(H\middle|\mathbf{d}_{\ell k}\right)}$$

Normalizing constant

- **Prior** from variogram and nearby data $\mathbf{d}_{lk}$
- **Likelihood** from seismic mismatch
- Get the **posterior** by sampling many **t**
- **Normalizing** constant can be ignored

Truncated Gaussian Likelihood and Posterior

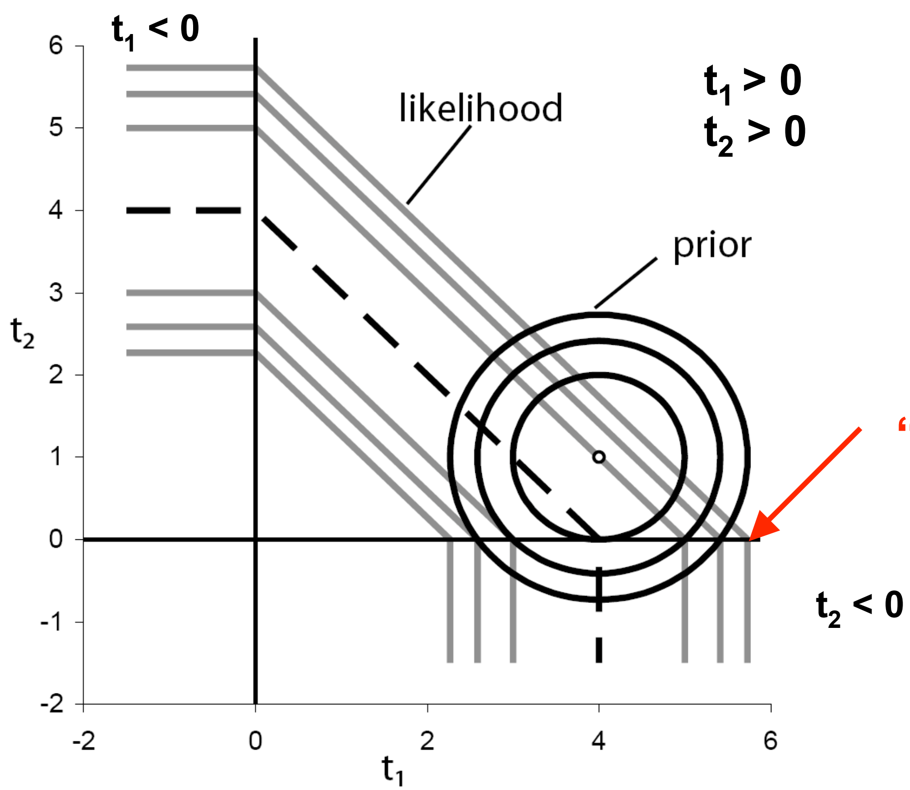
$$p(\mathbf{t}|\mathbf{d}_{\ell k}) \sim \boldsymbol{N}(\bar{\mathbf{t}}, \mathbf{C}_p)$$

$$p(H|\mathbf{t}, \mathbf{d}_{\ell k}) \sim \mathbf{N}_k(\bar{H}, \sigma_H)$$

Posterior Covariance

$$\mathbf{C}_\pi = \left[\mathbf{C}_p^{-1} + \mathbf{T}\mathbf{T}^T/\sigma_H^2\right]^{-1}$$

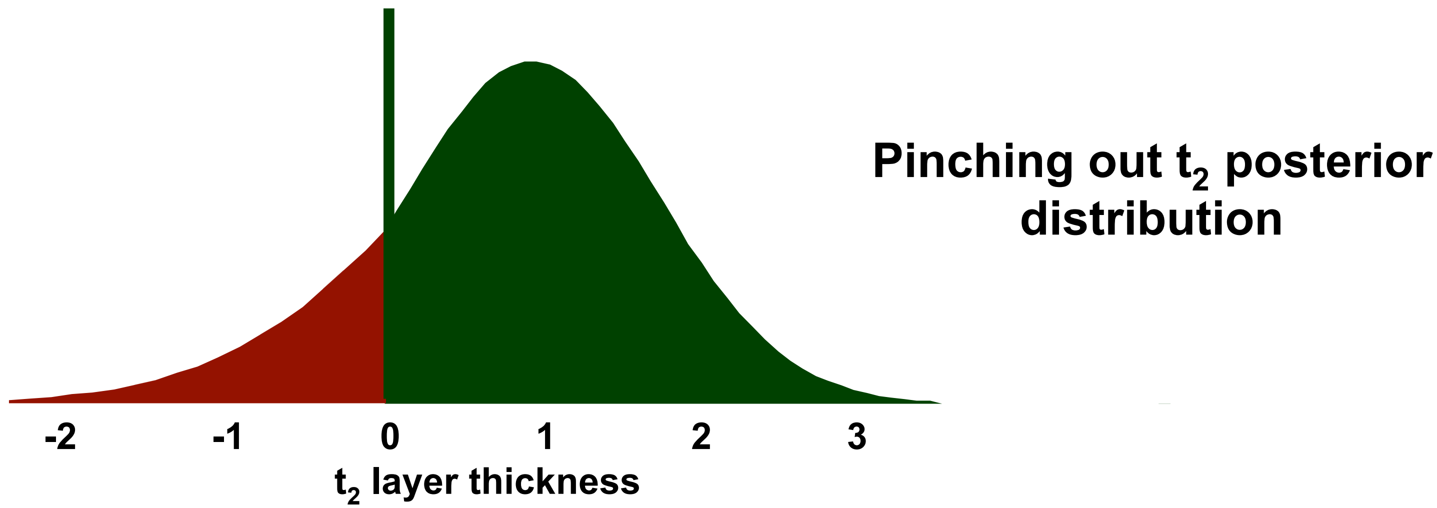
$$T_k = \begin{cases} 0 & \text{if } t_k < 0 \\ 1 & \text{otherwise} \end{cases}$$



"Stiff" nonlinear problem

t is a Gaussian proxy for h

Handling Pinchouts



- A Gaussian model is efficient and simple, but some of the proxies are negative
- For building geomodels set the thicknesses with negative proxies to zero

Truncated Gaussian Markov Chain Monte Carlo (TG-MCMC)

- Define auxiliary variable $u_{\mathrm{i}} = \{0, 1\}$ as indicator of truncation, 1 for $t_i > 0$
  - Treats “configurational stiffness”
- Plausible truncations by Gibbs sampling
- Metropolis transition probability for **t** includes thickness and auxiliary terms

$$\alpha = \min\left(1, \frac{\pi\left(\mathbf{t}'|H, \mathbf{d}_{\ell k}\right) \prod_{k=1}^{K} \pi(u_k|t'_k)}{\pi\left(\mathbf{t}|H, \mathbf{d}_{\ell k}\right) \prod_{k=1}^{K} \pi(u_k|t_k)}\right)$$

- Equivalent to sampling from the posterior

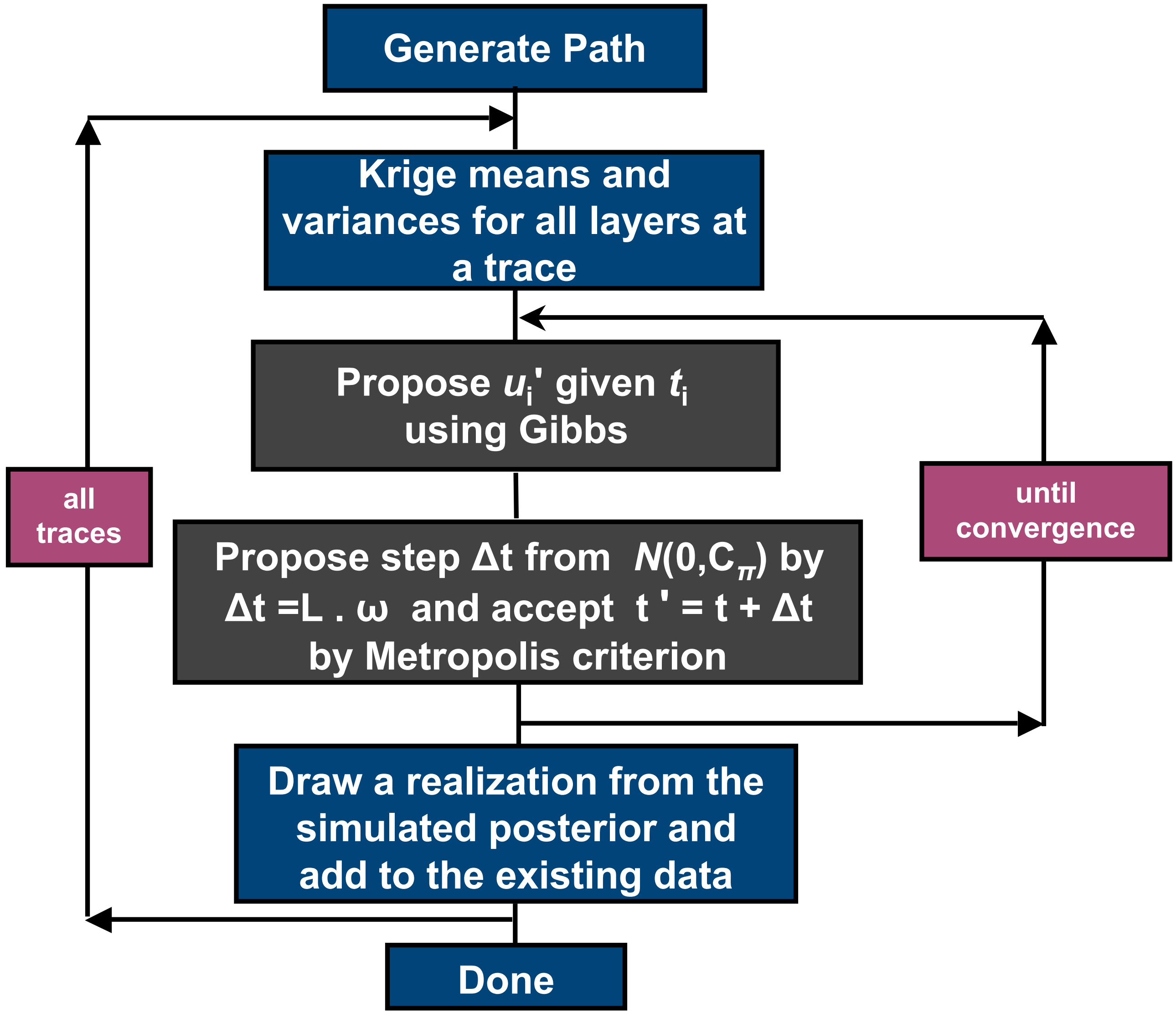
Assumptions and Performance

- Layer thicknesses are vertically uncorrelated at each trace
- Lateral correlations are identical for all layers
- Toeplitz form for resolution matrix

$$\mathbf{G} = \begin{pmatrix} \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} & \frac{1}{\sigma_H^2} & \cdots & \frac{1}{\sigma_H^2} \\ \frac{1}{\sigma_H^2} & \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} & \cdots & \frac{1}{\sigma_H^2} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{1}{\sigma_H^2} & \frac{1}{\sigma_H^2} & \cdots & \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} \end{pmatrix}$$

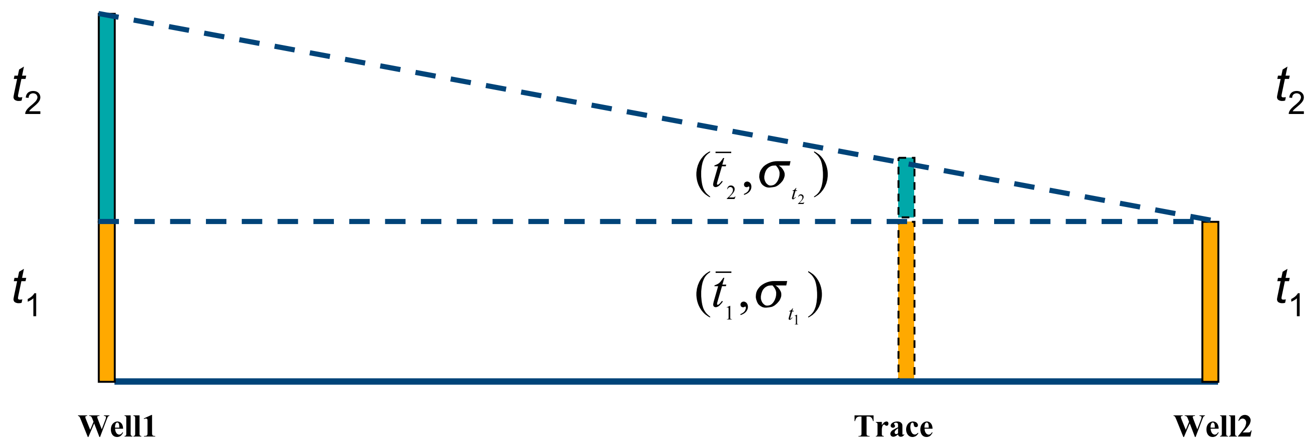
- Efficient Toeplitz solver
  - Handles layer drop-outs or drop-ins without refactoring

Sequential TG-MCMC

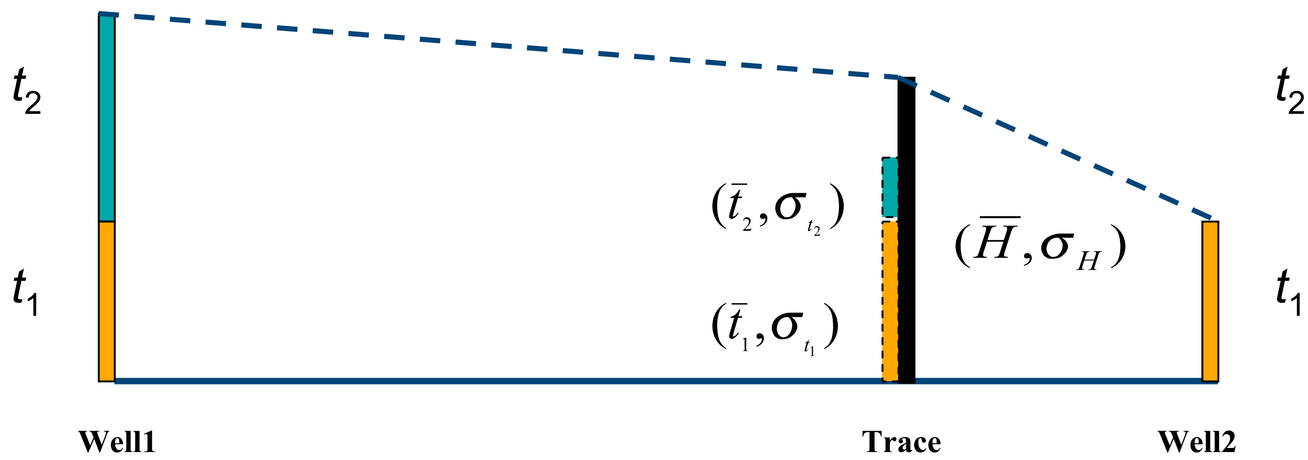


2D Examples

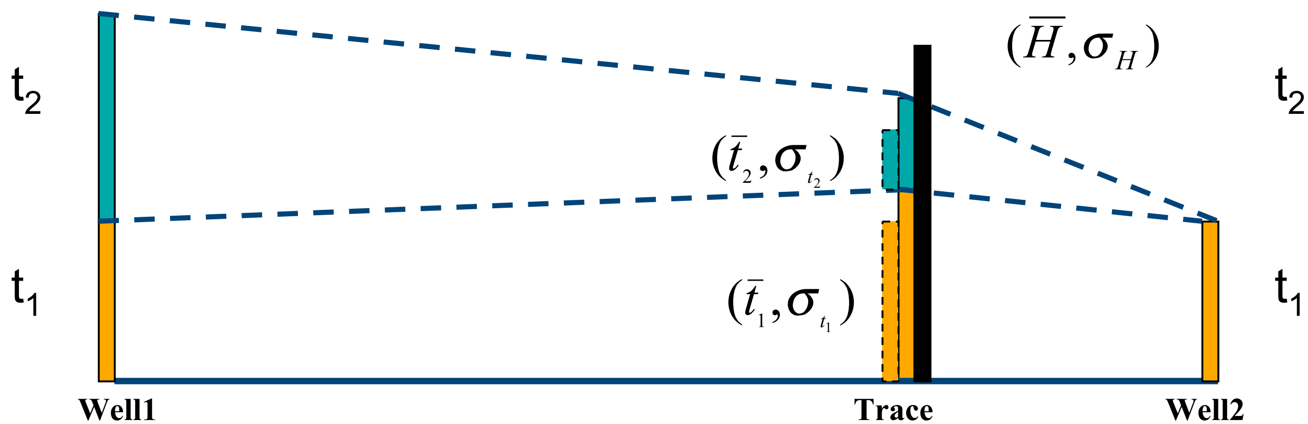
A Simple Two Layer Case



A Simple Two layer Case



A Simple Two layer Case



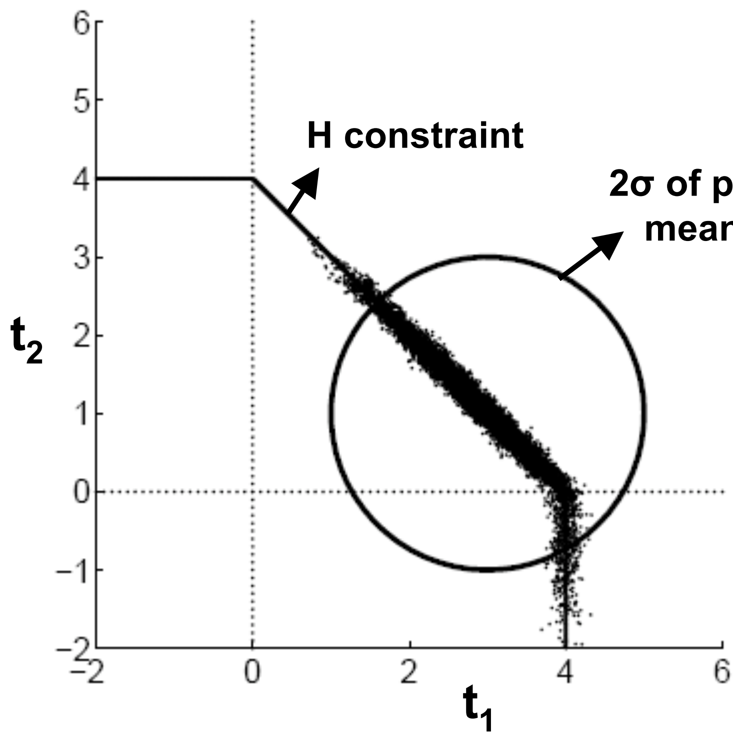
- Bayes reconciles seismic and well/continuity data
  - Posterior covariance weights each data type appropriately
- Simulation retrieves the complete distribution, not just the most likely combination

Spatial mismatch

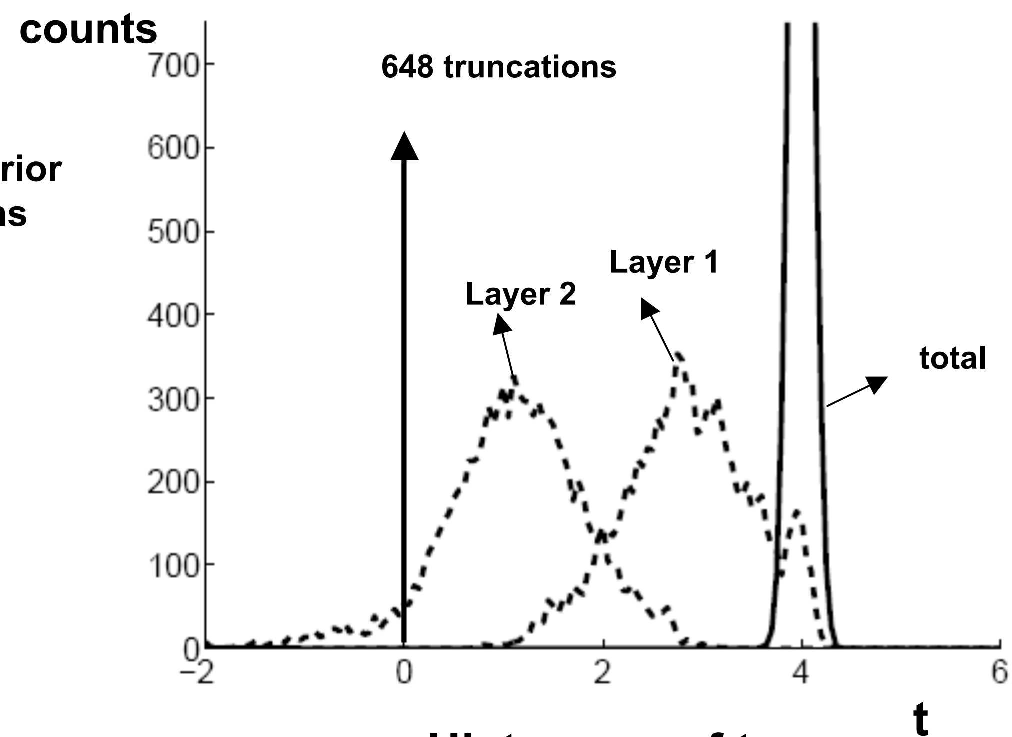
Seismic mismatch

$$\mathbf{C}_\pi = \left[\mathbf{C}_p^{-1} + \mathbf{T}\mathbf{T}^T/\sigma_H^2\right]^{-1}$$

Pinching Layer with Tight Sum Constraint



Scattergram, N=8000

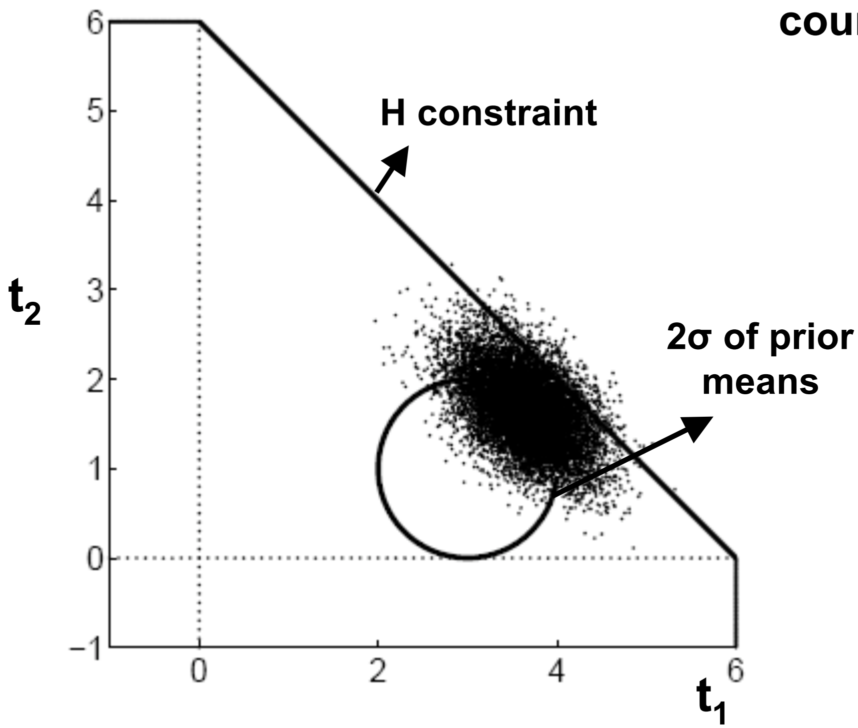


Histogram of t

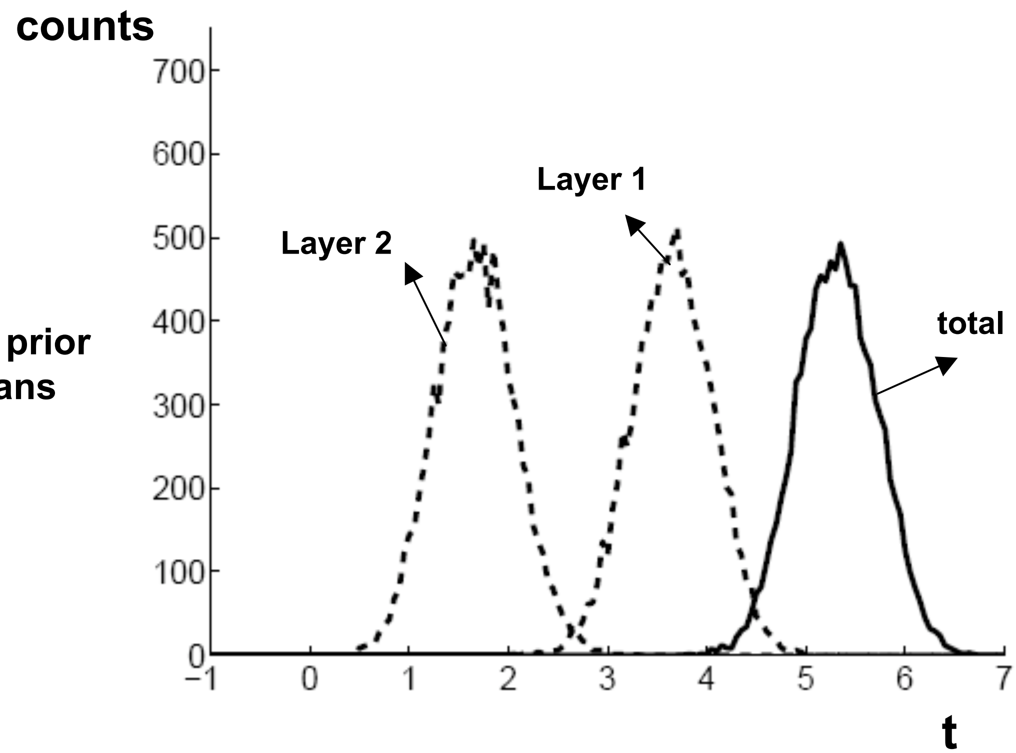
$$\overline{H} = 4\text{m},\ \sigma_H = 0.1\text{m}$$

$$\bar{\mathbf{t}} = (3\text{m}, 1\text{m}),\ \sigma_t = 1\text{m}$$

Prior sum not equal to Constraint



Scattergram, N=8000



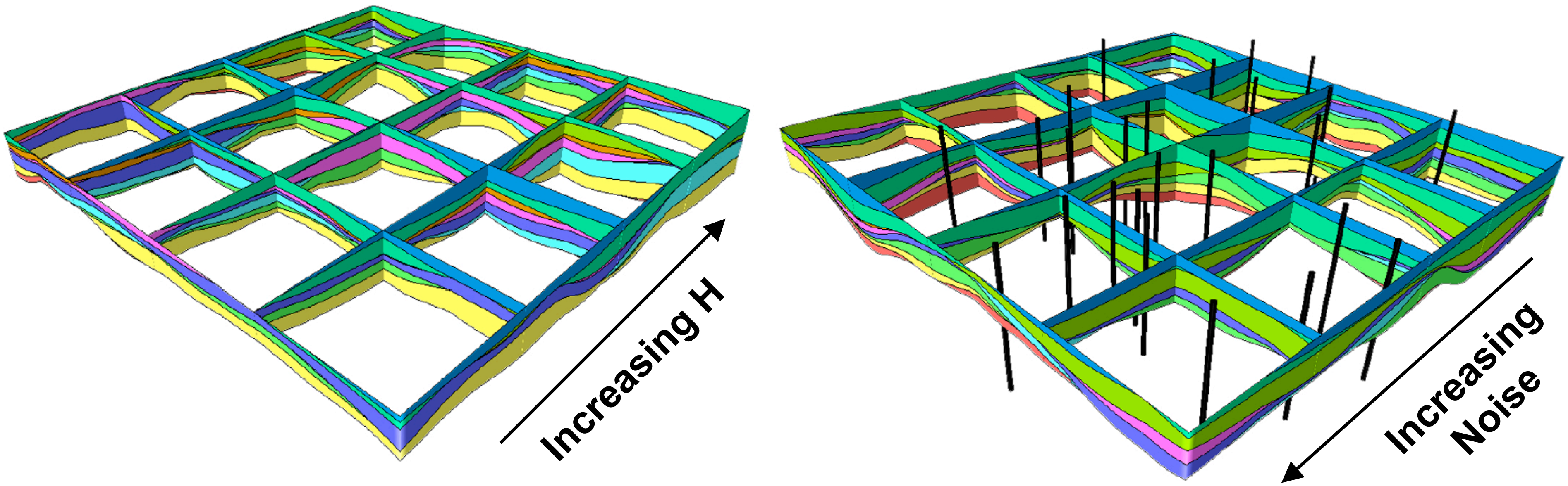
Histogram of t

$$\overline{H} = 6\text{m}, \ \sigma_H = 0.5\text{m}$$

$$\bar{\mathbf{t}} = (3\text{m}, 1\text{m}), \ \sigma_t = 0.5\text{m}$$

3D Examples

3D Problem :Trends

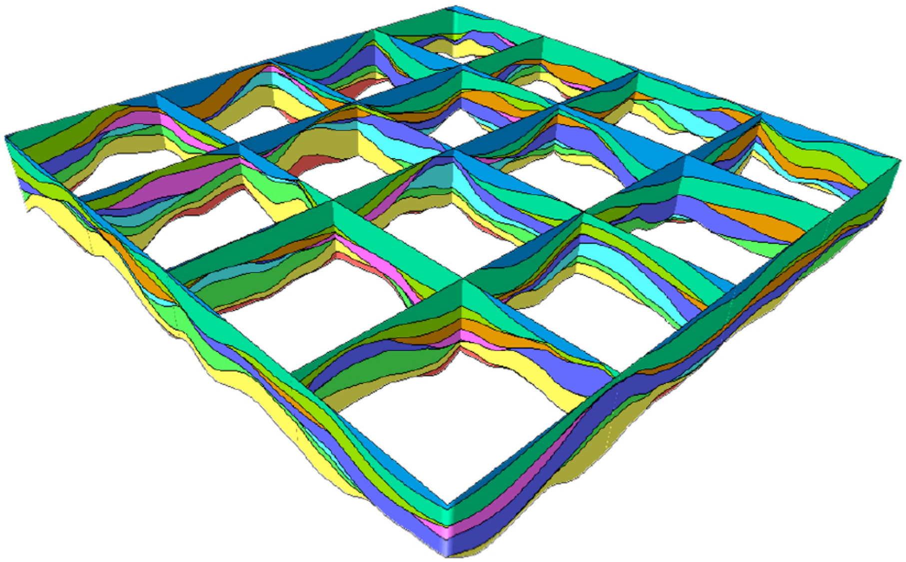


(a) Trend in seismic thickness, H

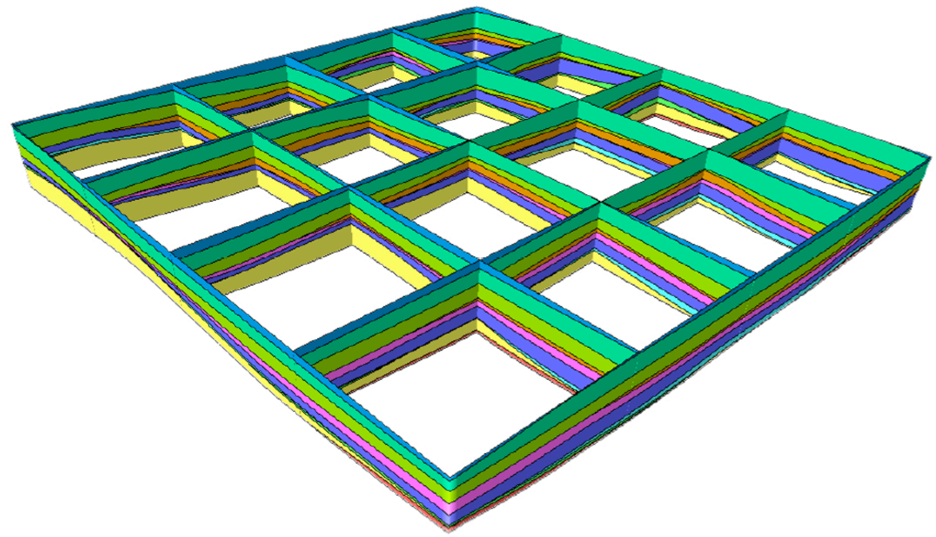
(b) Trend in seismic noise σ_H; same H trend as (a)

Simulations on a 100 x 100 x 10 cornerpoint grids with 25 conditioning data

3D Problem :Different Ranges



Short Range



Long Range

$$\overline{H} = 20\text{m}, \sigma_H = 2\text{m}$$

Performance Summary

Process	Work in Seconds
Kriging Work	5.95
Toeplitz solver work	0.22
Overhead for all 10^4 traces, 10 layers per trace	6.17
5000 samples, all traces	299.20
Total cost of simulation	305.37
Using 2 GHz Pentium-M processor with 1 GB of RAM *Implemented in ANSI C, g77 compiler, using NR & LAPACK routines*	

- 5000 samples for 10^5 unknowns in 5min on a laptop
- 98% of computation is for generating and evaluating steps
  - Toeplitz solve is almost free
- Fewer samples could be used in practice

Conclusions

- TG-MCMC consistently downscales seismic inversions and integrates well and variogram data
- Auxiliary variables model truncated layers
- TG-MCMC is adequately efficient with Toeplitz assumptions
- Extensions for exact constraints and other properties seem feasible

Acknowledgements

- Authors thank BHP-Billiton for funding this research with a gift
- Reservoir modeling software is provided by Schlumberger

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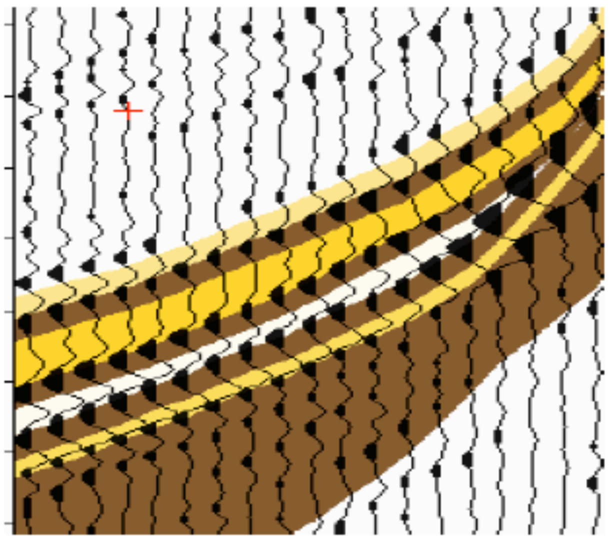
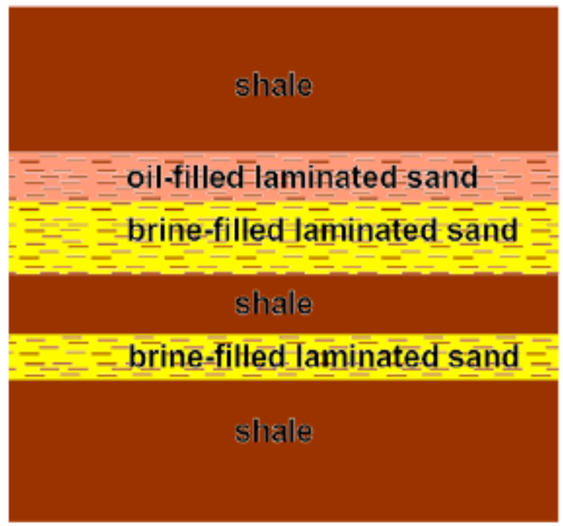
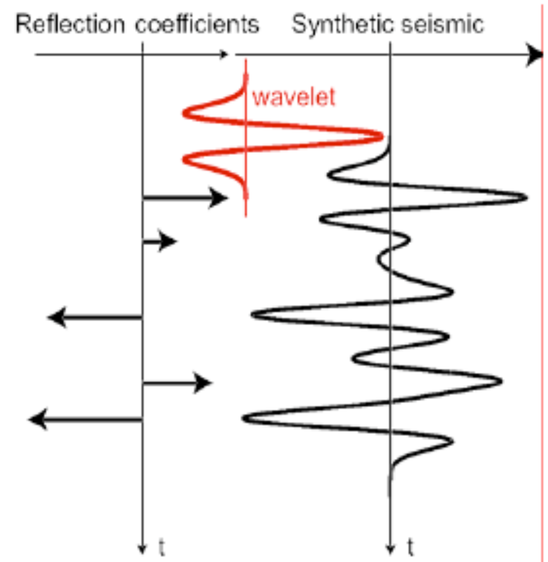
Delivery: Seismic Processing and Inversion Software

- Bayesian preprocessing (Gunning et al 2003, 2004)
  - Wavelet extraction
  - Time to depth maps
  - Well ties
- Bayesian seismic inversion code (Gunning et al 2005)
  - Set of plausible coarse scale reservoir models that honor seismic
  - Cornerpoint grid formats for reservoir simulation
- Bayesian methods help integrate diverse, uncertain data

Delivery Seismic Inversion







Traces | At a trace | Models

Gunning et al 2006

- MCMC Samples from posterior distribution
  - $\pi(t, V_{p,} V_{s,} \varphi, NG, \text{Fluid Type}, \ldots)$ for each layer

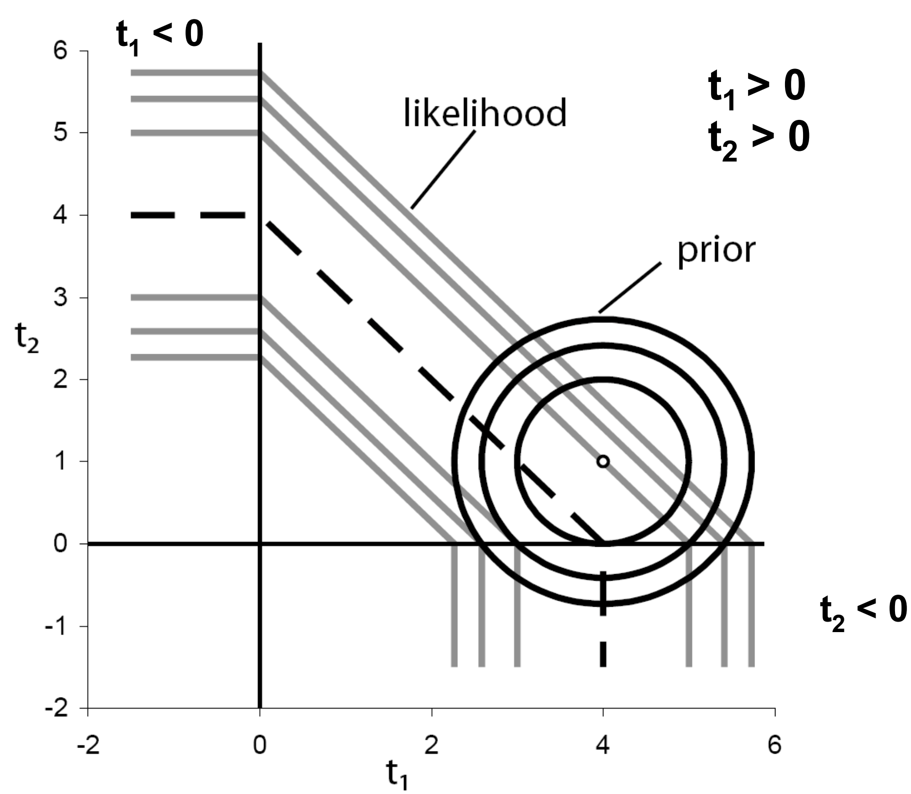
Truncated Gaussian Likelihood and Posterior

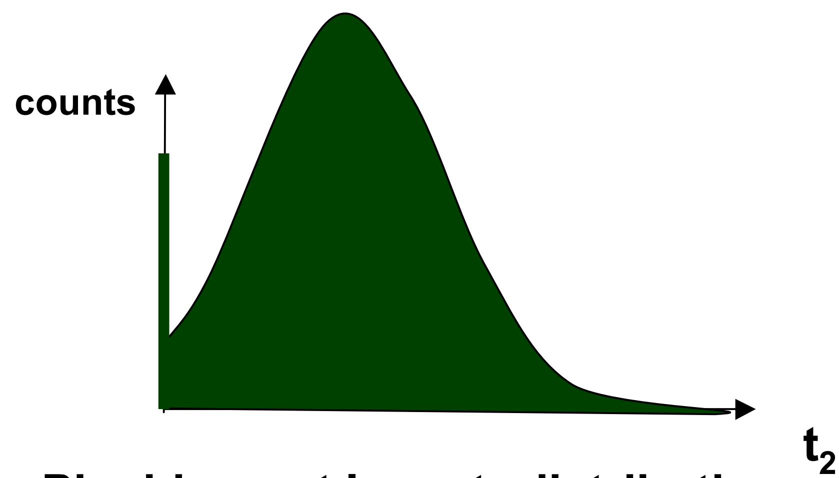
$$p\left(\mathbf{t}|\mathbf{d}_{\ell k}\right) = \frac{1}{\sqrt{(2\pi)^K |\mathbf{C}_p|}} \exp\left[-\frac{1}{2}(\mathbf{t}-\bar{\mathbf{t}})^T \mathbf{C}_p^{-1}(\mathbf{t}-\bar{\mathbf{t}})\right]$$

$$p(H|\mathbf{t},\mathbf{d}_{\ell k}) = \frac{1}{\sqrt{2\pi\sigma_H^2}} \exp\left[-\frac{(\mathbf{t}^T\mathbf{T}-\bar{H})^2}{2\sigma_H^2}\right]$$

$$\mathbf{C}_\pi = \left[\mathbf{C}_p^{-1} + \mathbf{T}\mathbf{T}^T/\sigma_H^2\right]^{-1}$$

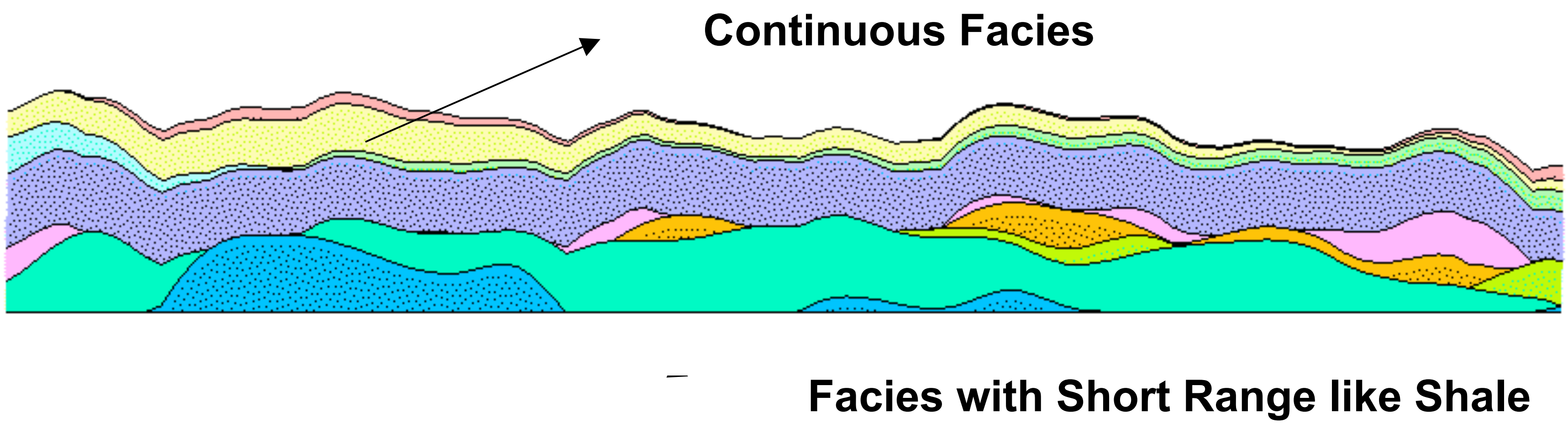
$$T_k = \begin{cases} 0 & \text{if } t_k < 0 \\ 1 & \text{otherwise} \end{cases}$$





Pinching out layer t_2 distribution

Multi Facies Modeling



- Facies with different continuity can be sampled independently as there is no vertical correlation
  - need (H_f,σ_{Hf}) of individual facies
- Here two different facies are included
  - top 5 layers are highly continuous layers (large range)
  - bottom 5 layers have short range

Sampling when Seismic Constraint is Tight

- Only *K*-1 degrees of freedom are available as

$$\sum_{i=1}^{K} t_i = H$$

- Construct a new *K*-1 dimensional orthogonal basis using Gram-Schmidt or SVD
- Sample on this new basis **t**'
- Need to build (unique) transformation matrix *U* mapping to original coordinates **t** =*U* **t**'

Ongoing Research

- Several Distinct Facies inclusion in each seismic loop
- Sampling on the constraint hyperplane
- Implementation of Block Methods to address the concerns with sequential methods
- Constraint on porosities and other nonlinear properties
- Selecting Realizations by upscaling the properties, simulating, and principle component analysis (PCA)

Markov chain Monte Carlo (MCMC)

- Samples from posterior using Markov and Monte Carlo properties
- A Markov Chain is a stochastic process that generates random variables $\{ X_1, X_2, \ldots, X_t \}$ where the distribution

$$P(X_t \mid X_1, X_2, \ldots, X_{t-1}) = P(X_t \mid X_{t-1})$$

  i.e. the distribution of the next random variable depends only on the current random variable

- These samples can be used to estimate summaries of the posterior, π, e.g. its mean, variance.

Data Augmentation : Handles bends in the posterior

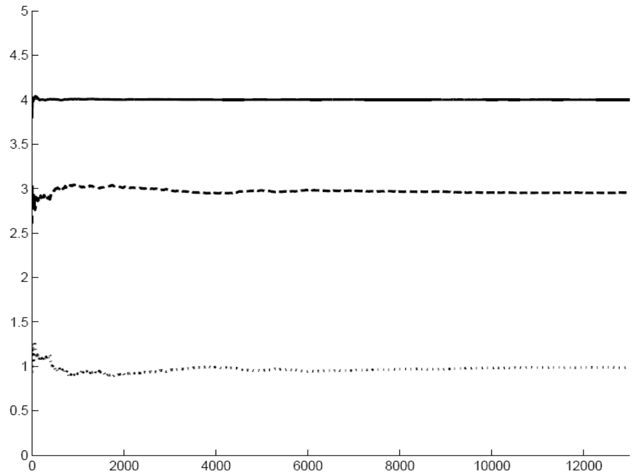
- Reversible MCMC hopping scheme that adjust to the proposals to the shape of local posterior
- Define auxiliary variable u_i={0,1} as indicators of the layer occurrence
- Sampling in indicator space is done by Gibbs sampling
- This handles pinchouts; details of **t** are handled in a Metropolis step

Metropolis for **t**

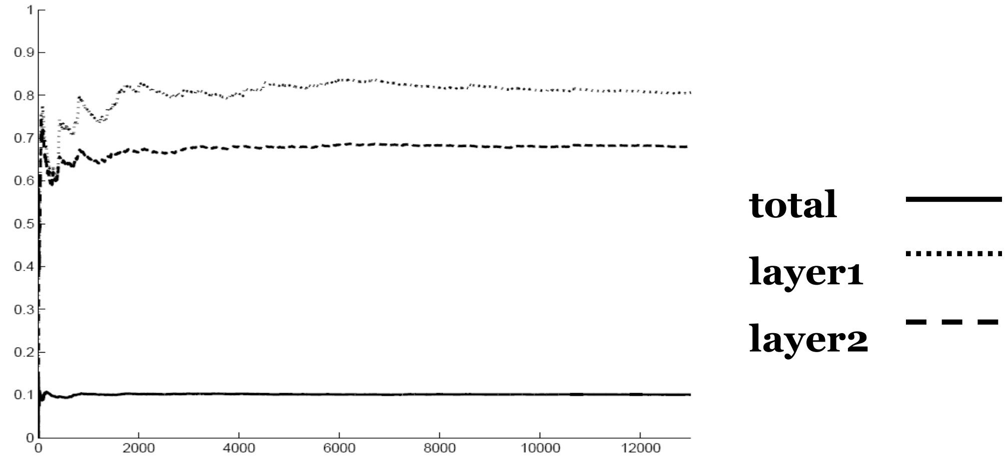
- It is possible to construct a Markov Chain that has the posterior as its stationary distribution
- In the current step, the value of the parameters is X_t. Propose a new set of parameters, Y in a symmetric manner.
- Calculate the prior and likelihood functions for the old and new parameter values. Set the parameter values in the next step of the chain, X_{t+1} to Y with probability α, otherwise set to X_t

$$\alpha(X,Y) = \min\left[1, \frac{\pi(Y)}{\pi(X)}\right]$$

Convergence of Mean and Variance



Mean Convergence



Std Deviation Convergence

- Should converge to target distribution in as few steps as possible
- Hopping
  - large steps $\rightarrow$ acceptance rate low
  - small steps $\rightarrow$ don't explore posterior
  - Scaled posterior $\rightarrow$

$$\tilde{\mathbf{C}}_\pi = \frac{5.67}{K}\mathbf{C}_\pi$$