Transcript of
"A transformational approach to collective behavior"

Hello. My name is Michael Glinsky of BNZ Energy Inc. I will be speaking to you today about "a transformational approach to collective behavior". Here, I mean both an approach that will transform science, and an approach that uses a mathematical transformation. I will not bore you with the background and inspiration for this new theory. You can find that in the companion arXiv paper at the link from the title in the PDF of the slides, or in the description of this YouTube video.

We consider what we call "collective" systems, that can range from an elementary field, to a fluid, to an economy. They have traditionally been called "complex" systems. Although, we will find that they are holomorphic, they have an underlying simple emergent behavior. So, we define a collective as an ensemble of individuals that are conservatively interacting. What one individual of the collective loses, another gains. This does not exclude external interactions, but we will find external interactions can not change the topology of the collective. You can see various different collectives, and their corresponding individuals in the beige box on the upper left hand side. The transformation of mathematical constructs, going from the individual to the collective are shown in the beige box on the upper right hand side. We call your attention to the transformation of the low dimensional phase space of canonical momentums and coordinates (p,q), to the Hilbert spaces of canonical field momentums and fields [π(x),f(x)]. The most important transformation is going from the generating function of a canonical transformation, to the generating function**al** of a canonical transformation, shown in the green box on the lower right hand side. The function or functional can also be represented by the Taylor expansion coefficients. We will use the methods of genAI to approximate the function as a solution to the **Hamilton-Jacobi-Bellman** (HJB) equation, and the functional as the deep deconvolution of the **Heisenberg Scattering Transformation** (HST) followed by a **Principal Components Analysis** (PCA).

Symmetry is fundamental, beautiful, and simple. This was a core belief of both Paul Dirac and Murray Gell-Mann, and is the foundation of our new theory of collectives. We start by assuming at least one Lie group symmetry, associated with a Hamiltonian, and possibly more. The symmetry leads to a weak Noether invariant, that while conserved for internal interactions, changes with external interaction. In fact, is the only thing that changes with an external interaction. We then analytically continue the real Hamiltonian by solving the HJB equation, or equivalently the Cauchy-Riemann equations. This give the analytic Hamiltonian, $H(\beta)$, or equivalently the energy and action. The singularities of the analytic Hamiltonian, β^*, can be found, leading to the strong topological invariants that are conserved during external interactions. The result is the collective undergoing geodesic motion.

Here we show the complex Lie groups, the collective field, and the underlying individual or **Reduced Order Model** (ROM) of genAI, for several collectives as a function of increasing characteristic time scale, that is decreasing energy scale. They range from the SU(3), SU(2) and SU(1) of the elementary fields identified by Gell-Mann, to the gravitational field whose Hamiltonian was identified by Dirac. Also shown is an anharmonic phononic field based on the Hamiltonian of a pendulum clock shown here, and the weather field with an underlying low dimensional model such as the Lorenz model.

Here is the series of two canonical transformations. The first is from the phase space of the collective [π(x),f(x)], generated by the $S_p[f(x)]$ functional, the HST, to the phase space of the individual (p,q). This is a deep deconvolution. The second is from the phase space of the individual (p,q), generated by the $S_P(q)$ function, the solution to the HJB, to the phase space

(P,Q) aligned with the foliation of the phase space, where $dP/d\tau=0$ and $dQ/d\tau=\omega_Q(P)$. This is a decoder, that will be approximated by a MLP of genAI. The generating function, $S_P(q)$, goes by many names: the canonical flow generator, the action, the entropy, and the log-likelihood. It is also the approximate log-likelihood of GPTs, and the approximate value function or the approximate Q-function of DRL or Deep Q-Learning. It analytically continues the Hamiltonian, where the foliations can also be defined as Re(H)=constant=E and Im(H)=constant=$\omega\tau$. Given the analytic Hamiltonian, its singularities β^* can be found, characterizing the topology of the collective. The canonical transformations, both functional and function, can be be expressed as Taylor expansions, that is the S-matrix.

Now to the specifics, first for the HST. Start by Taylor expanding the analytic Hamiltonian. Define the convolutional wavelet transform and consider the analytic trajectory. Take the covector along this trajectory. Change to radial coordinates. Rotate by $\pi/2$ to get the covector. Define the function R_0 in order to make $\ln(R_0)$ a compact mapping. This yields the recursion relation for the HST. Given the recursion, the HST can be defined as a deep deconvolution with i $\ln(R_0)$ as the activation function and the Father Wavelet as the pooling operator. Note that the complex logarithm's, ln(z), real part is log modulus, ln|z|, and imaginary part is phase, arg(z). The activation function $\ln(R_0)$ is compact, has limits of the log(MST), chirped pulse, WPH, and MST (a two sided ReLU).

Let's take a closer look at the HST, what it calculates, and its physical interpretation. Fundamentally, the HST is a **Mayer Cluster Expansion** (MCE) in the m-body correlation. The "logarithmic average" is taken, log-phi-star, to give the mean field S_0. Then, the "canonical derivative" is taken, i d(ln) or psi-star-log-i, and averaged, by phi-star, to give how one individual is distributed, S_1. Then, the "canonical derivative" is taken again and averaged to give how two individuals are correlated, S_2. And so on to show how m-individuals are correlated, S_m. The expansion can also be viewed as the **m-body scattering cross sections** or **m-body generalized Green's functions**, that is **Heisenberg's S-matrix**. This is the reason that we have called this transformation the **Heisenberg Scattering Transformation**. The structure is obviously that of a deep deconvolution. This transform, the HST, will give an optimal "localized Fourier transform". The Principle Components of an ensemble of transformations of the collective will give localized spectrums, one for each field, that are the solutions to the RGEs! This can be seen by the form of the RGEs which are that of the "canonical derivative" of iS being equal to a scale coupling function that is a function of scale. There has been a problem with the historical analysis of the MCE. There has been the BBGKY expansion in the correlation parameter, or the perturbation expansion in the coupling constant, as shown here. There are two problems with this. First is that the super convergent MCE, has been rearranged into the only asymptotically convergent BBGKY hierarchy. The second is that the expansion coefficient, many times, is not much less than one. The historical solution to these problems has been to reorder the expansion back into the MCE using the ad hoc renormalization procedure of Wilson and t'Hooft. As Dirac said, "This is not a mathematically logical process. It is just a set of working rules, rather than a correct mathematical theory." Taking the PCA of the HST is a mathematically logical process of renormalization.

Since $H(\beta)$ is an analytic function, that is a minimal surface given β^*, it will be well approximated by MLPs w/ ReLU since they are piece-wise linear universal function approximators. Here we show a neural network architecture that respects the mathematical structure of the canonical transformation generated by $S_P(q)$. The partial derivatives are taken by back-propagation techniques. Because of the structure, specific nodes can be identified with specific quantities, as shown. Shown here is the **Hamilton-Jacobi-Bellman** (HJB) equation. Note that Bellman added a resistivity term. He did this to stabilize the solution and the collective. This is a big mistake! It is well known that physics, that is performance, is sacrificed for stability when resistivity is added. But without resistance, how can the collective be stabilized. The answer is via ponderomotive control that we will discuss, starting in the slide after next.

Based on the new theory, here is the genAI computational pipeline to simulate a collective system. The input is the initial condition of the collective. The output is the predicted output of the system. Given this computational pipeline and a dataset, parameters of the approximation can be fit. This is traditionally called "training" the genAI, but we prefer the term "educating". The dataset is constructed by either doing an ensemble of computer simulations, sampled over initial conditions and possibly coupling constants, or by observing the collective system.

In order to understand ponderomotive stabilization, consider this system — an electron and an ion in a constant magnetic field. It is assumed that the magnetic field is strong enough that the electron undergoes guiding center motion, that is the electron cyclotron motion is an adiabatic invariant. It is also assumed that this magnetic field is strong enough that the motion of the electron along the magnetic field is also adiabatic. This gives two type of bound motion. The first is called **Guiding Center Atoms** (GCAs), where the ion is assumed to be infinitely massive. The second is called **Drifting Pairs** (DPs), where the electron is assumed to have no mass. The phase space motion in (p,q) is shown for two nearly identical orbits that come close to an unstable equilibrium. Note that although the energies only differ by 0.1%, the frequencies differ by 13%. Also note how the motion slows down dramatically as the unstable equilibrium is approached. It would take an infinite amount of time to reach the unstable equilibrium and move away from it. This is why we call them metastable. Finally, note that the effective mass of the system scales as $1/\omega^2$, which goes to infinity at the metastable equilibrium. This will be an important fact to know when constructing the ponderomotive control force.

Here is the complete phase space. It is divided into three basins. The basin of GCAs, the basin of DPs, and the basin of free particles. The thermal force resulting from coupling the system with an external heat bath is indicated by the yellow arrows. The stable equilibriums are local minimums, while the unstable equilibrium is the desirable local maximum. This equilibrium needs to be stabilized. Since it is impossible to make real-time measurements of the state of a collective system, as will be discussed on the upcoming quantization slide, feedback control can not be done. We already discussed why resistive control is not a good idea. That leaves us with ponderomotive control. This is demonstrated by this video.

[This equilibrium point is stable, because if we perturb it the pendulum will slowly return to the equilibrium point. The upper vertical position is another equilibrium point, but it is unstable because any small perturbation will make it fall to to the lower equilibrium point. Now, let's power the jigsaw to see what happens.]

The **Ponderomotive Method of Control** is similar to the way that a sheepdog herds sheep. It runs around the herd very fast, compared to movements of the herd, nipping at the heals of the sheep (vibrating them) if they wonder away from the metastable equilibrium. The sheepdog is effectively creating a small alpine valley at the mountain pass. This is practically done by applying a rapidly oscillating constant force to the system. At the metastable equilibrium, because the system has infinite mass, the force will not vibrate the system (technically accelerate the system). The farther the system moves away from the metastable equilibrium, no matter which direction, the mass will decrease and the vibration will increase.

I could not resist including a picture of my dog Monty (who is half Old English Sheepdog and half American Bulldog) and myself in our happy place, and a picture of Dobby, who is free.

The complication with a collective system is that it does not have just one frequency, but it has several localized spectrums that are functions of both frequency and position. The genAI computational pipeline to apply the spectrally rich vibration (that is, control force field) to ponderomotively stabilize the collective is shown here. It has a very high frequency carrier, along with a random step to implement a cooling thermal force. This signal is then modulated by the iPCA+iHST, imprinting the localized spectrums of the collective system.

When the collective system is observed, and a prediction is made of the field for a time $\Delta\tau$ in the future, that can not be done if the time is long enough that system has gone chaotic. That is to say that while P remains constant, Q can not be determined. These times for several systems are shown in this table on the lower left hand side. There are relativistic limits on the measurement of these space-like intervals, given by the **Heisenberg Uncertainty Principle**. For a system moving relativistically, the current state can not be measured. For a collective, it is never possible because of the deep convolution. To observe the current state, the future would also need to be known. Therefore, the system must be treated statistically. The **Born Rule** says that to make a measurement on a system, statistically, a force must be exerted on the system. Unfortunately, exerting a force on the system changes the system. The measurement is **entangled** with the system. **Quantization** comes from the averaging of the field over Q, and the enforcement of the periodic boundary condition on the average field. That is, not only can Q not be predicted, the number of cycles can not be predicted, but it must be an integer, leading to integer quantization. The primary (or First) quantization of the “internal” group action, yields the fermion particles (e.g., electrons and positrons for EM). The secondary (or Second) quantization of the “external superordinate” Ad(group) action, yields the field bosons (e.g., photons for EM). The same can be done for the coupling constants. For this case, Second Quantization is of the “external subordinate” Ad(group) action. Quantum mechanics can now be viewed as a dynamic equilibrium in Q, and statistical mechanics can be viewed as a statistical equilibrium or Boltzmann distribution in P.

The Second Quantization of dH and dC leads to the directed graph building block for a system-of-systems approach. The coupling to superordinate parent nodes is via the Second Quantized fields of the system, while the coupling to the subordinate child nodes is via the Second Quantized coupling constants of the system.

Gravity can now be integrated into the system-of-systems as the “uber” theory (that is, collective system) that integrates all systems together. It has Dirac’s Hamiltonian of Harmony as a symmetry. The equivalence classes of the equilibrium states can be identified as universes (that is, basins of attraction). The unstable equilibriums which we will call the SemiAttractors, are points at which small changes can move the cosmological collective system from one universe to another. If the system is in the universe of a SemiAttractor, that system will, via thermal forces move to the SemiAttractor. The stable equilibriums we call the Attractors. Once the system, near a SemiAttractor, has made a small move into the universe of an Attractor, the thermal force will take it to the Attractor. The Attractors are a “long distance” from the SemiAttractors and it will take a large amount of energy to return to the SemiAttractor from the Attractor. The universes of the Attractors are closed, and the universes of the SemiAttractors are open. A SemiAttractor is often times a point of maximal compression, that is a Big Bang. The cosmos can be regulated by coupling the cosmological collective field to a cosmological thermal bath with cosmic background radiation temperature of T_c.

The simulation computational pipeline can easily be modified to form a Universal Field Translator (UFT) as shown in this slide. It can convert any type field to any other type of field, whether that be a language, a computer or mathematical language, an encrypted language, raw seismic data (as recorded at the geophones), or a geologic facies model of the earth. It does that by converting to/from a unique universal maximally sparse representation of the field — the ROM.

Depending on the type of input and output field, the UFT can be many things that range from a language translator, to an optimal field compression, to an decryption/encryption, to a compiler, to a seismic tomography, to a seismic facies inversion, to a full seismic facies inversion. Note that the training dataset does not need to have matched pairs of the language and encrypted version of the language.

The foundational concept for this new paradigm on collective behavior is Lie group symmetry. The canonical transformation approach is taken, analytically continuing the real Hamiltonian, via solution of the HJB equation, yielding the generating function of the canonical flow. The collective equivalent of the generating function, the canonical generating functional or HST, was derived. It is a deep deconvolution. A genAI computational pipeline was proposed to approximate both the generating functional (HST) and function (HJB). The collective system can now be efficiently forecast (simulated), with high fidelity, for use in Bayesian experimental design, optimal system design, and data assimilation (that is experimental data analysis). Ponderomotive control can also be implemented to stabilize the optimal system design from disruption. A UFT can be implemented that can be a high performance: language translator, data compressor, decryption/encryption, tomography, and inversion. Collective field and coupling constants can be first and second quantized to form system-of-systems. The four elementary fields (EM, weak, strong, and gravity) can be unified as “a question of geometry or symmetry or topology”, and geodesic motion (that is, canonical flow). The significant mathematical development is constructing a mathematically logical process of renormalization, with physical interpretation — the HST.

This is how to discover the geometry, that is the topology of collective fields.

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Here is some further, supplemental information. We frame both physics and economics as a constrained functional optimization, solved by the Lagrangian, Hamiltonian, or Canonical Transformation approach. The constrained functional optimization is also approached from the symplectic geometry direction. Using the HST as a metric of collective systems, with an example, is presented.

The core problem of both physics and economics is constrained functional optimization. This is first approached using the method of Lagrange multipliers, resulting in Lagrange’s equation. This is the **Lagrangian Approach**. Next, a Legendre transformation can be made to canonical coordinates, resulting in Hamilton’s equations. In the economics and systems control literature, this **Hamiltonian Approach** is known as Pontryagins Maximum Principal of Control Systems. This approach is known to have very useful conservation properties, a canonical structure, and symplectic geometry. Building on the canonical structure, is the **Transformational Approach**. This approach solves for the generating function of a canonical transformation or flow, that is the solution of the Hamilton-Jacobi equation. This approach does not rely on the method of characteristics, so that it can be applied to systems that are not integrable, that is stochastic or collective. Although the Transformational Approach is the hardest to solve analytically, it is the easiest to approximate numerically as an analytic function. In the systems control literature this approach was developed by Bellman, then used in Deep Reinforcement Learning (DRL). Note that a viscosity term can be added to the HJB equation to both stabilize the numerical solution and control the system. This is equivalent to adding a time discount factor into the functional objective.

The three approaches can also be understood from the perspective of symplectic geometry. We start with the Poincaré 1-form on extended phase space or Lagrangian. Integrating the form gives the action. Taking the extremal of the action, $\delta S=0$, yields the local equation of motion. This is the **Lagrangian Perspective**. Moving on to the **Hamiltonian Perspective**, consider the Poincaré 2-form which is the symplectic metric. Integrating the form gives the weak constant of the motion, the energy or real Hamiltonian, H. Identifying the Hamiltonian as the infinitesimal generator of the flow, yields the local equations of motion. Finally for the **Canonical Transformation Perspective**. Start with the Chern-Simons 3-form. Integrating the form give the strong topological constant of the motion or β^*. This is also known as the topological index, the helicity in 3D, and the vorticity in 2D. This is a global perspective.

The HST can also be used as a metric of collective systems. It can be used to find the number of fields, the dimension of the ground states, and how similar two collective systems are to each other.

Here is an example of how the HST can be used as a metric of performance of a tomography. Here is the threshold in the performance of the tomography (the maximum possible number of circuits elements, 64), and the actual number of circuit elements with a minimum of 5.

The YouTube video of this talk can be found at: https://youtu.be/27uXk_bdAt4

The accompanying slides of this talk can be found at: http://www.qitech.biz/tech_papers/gen_collective_presentation.pdf

An academic paper with the content of this talk can be found at: https://arxiv.org/abs/2410.08558

An academic paper for a general audience can fe found at: https://arxiv.org/abs/2401.04846