Relationship of the Mallat Scattering Transformation (a deep convolutional network) to causal physics and complexity

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The Mallat Scattering Transformation (MST) is a hierarchical, multiscale, transformation that has proved to be effective at distinguishing textures, sounds, written characters [1–3], and the emergent behavior (self organization) of magnetized Z-pinch implosions [4, 5]. It has also been shown to be a form of deep learning related to convolutional neural networks [6]. This talk will explore its meaning, its relationship to causal physics, and its significance in the analysis of complexity.

We have developed theory that connects the transformation to the causal dynamics of physical systems. This has been done from the classical kinetic perspective (using a coordinate free exterior calculus formalism [4]) and from the field theory perspective [7], where the MST is the generalized Green's function, or S-matrix of the field theory [8] in the scale basis. From both perspectives the first order MST is the current state of the system, and the second order MST are the transition rates from one state to another.

If one includes the evolution coordinate, that is time, in the transformation, the second order MST directly, and with no further transformation, gives the transition kernel of the dynamics. This is independent of the current state, that is the first order MST. Given an ensemble of example states that sufficiently sample the transition kernel, one has fully characterized the physical system and should be able to evolve any state forward in time, as given by the initial first order MST. That is the MST is the perfect coordinate system in which to learn, identify, and propagate the dynamics.

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