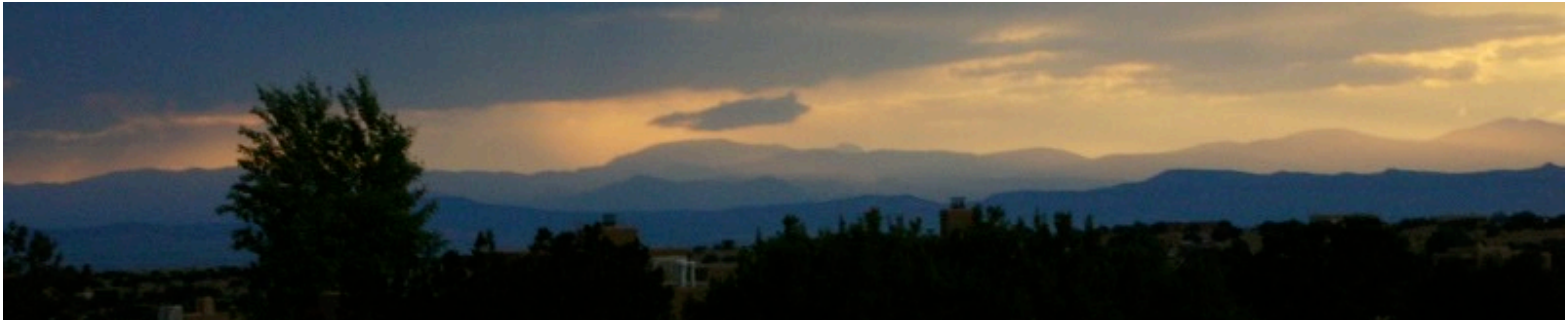

Field surrogate of Gorgon 2D liner implosions using the Mallat Scattering Transformation (MST)

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Kathryn Maupin, PhD

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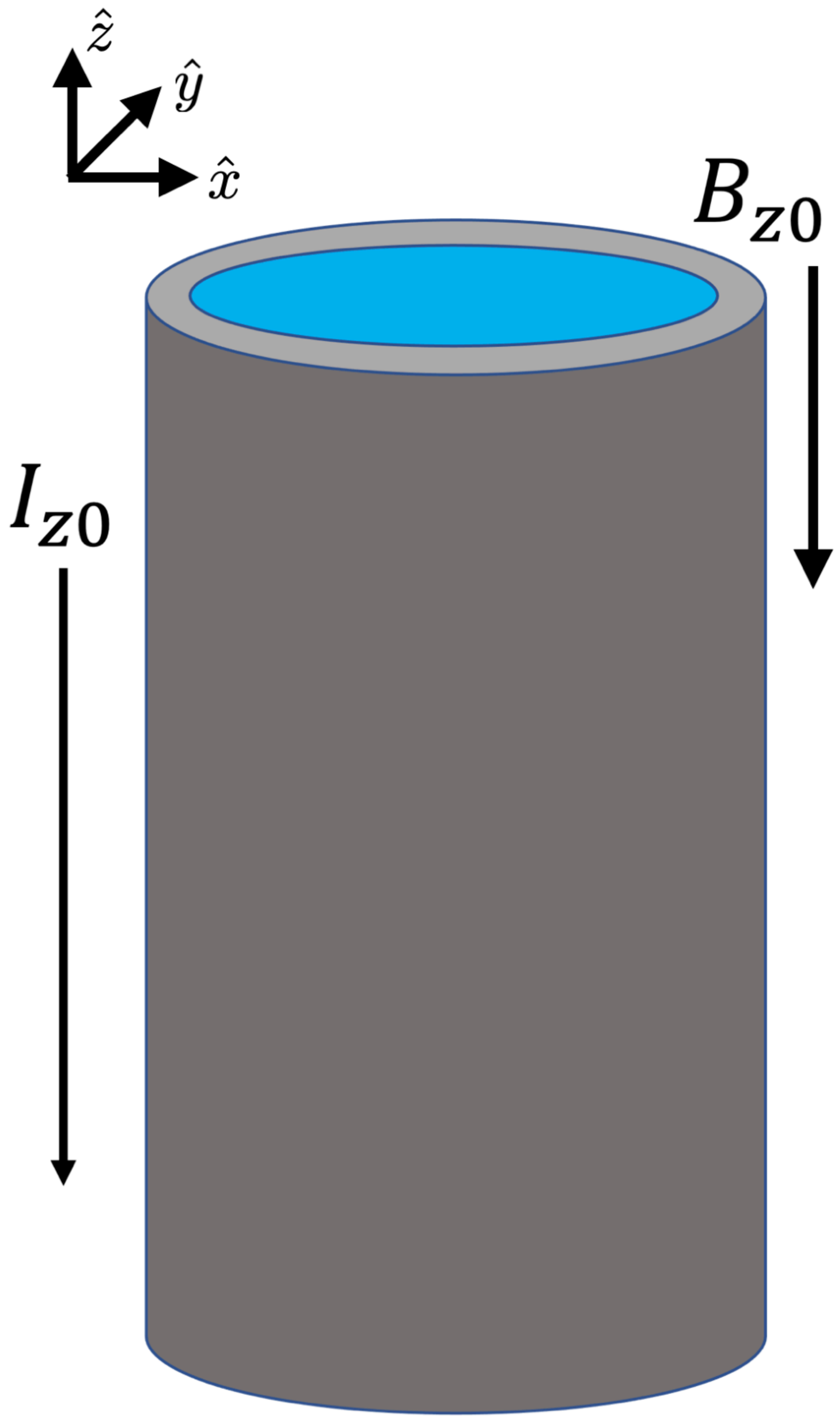
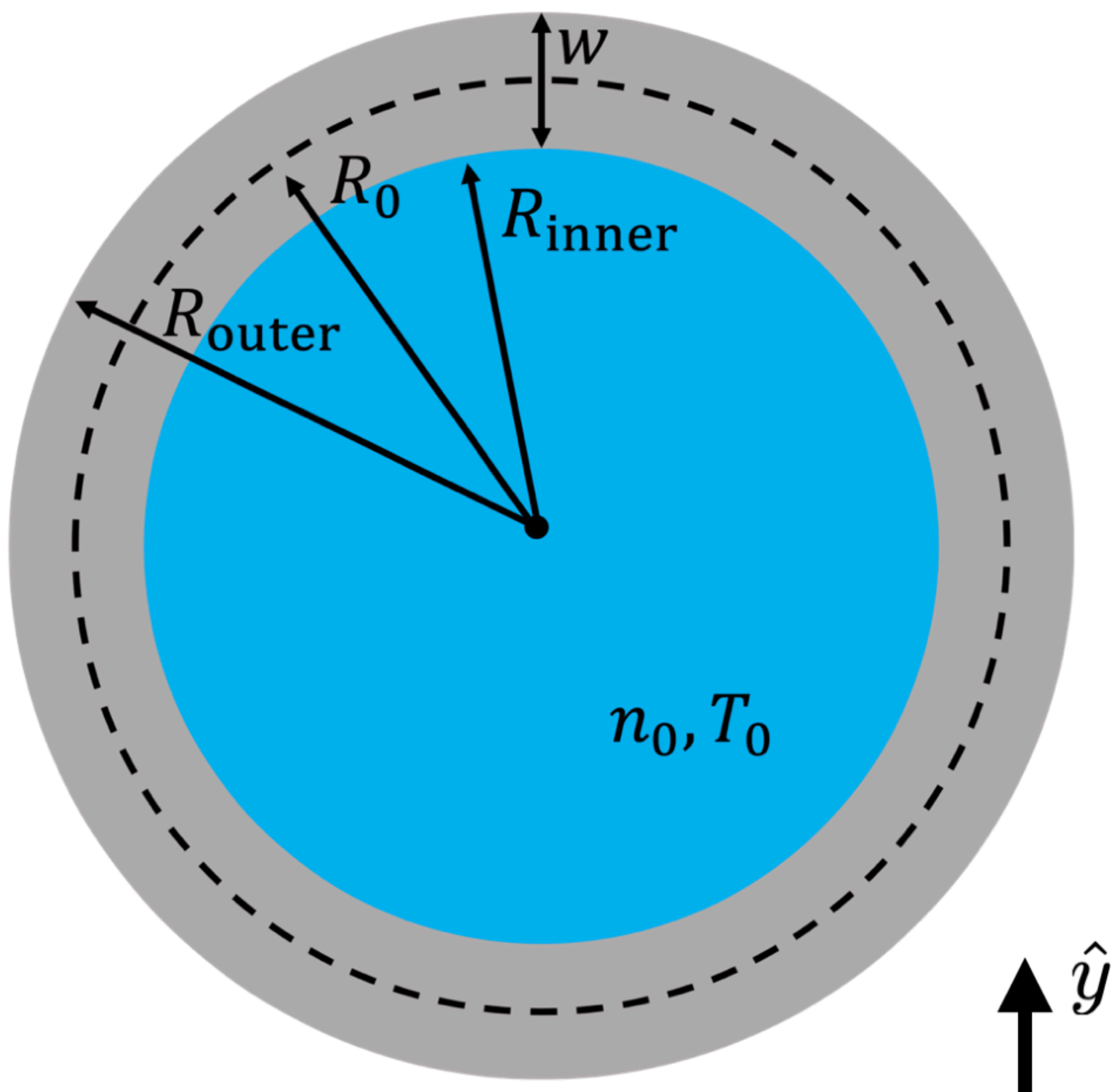
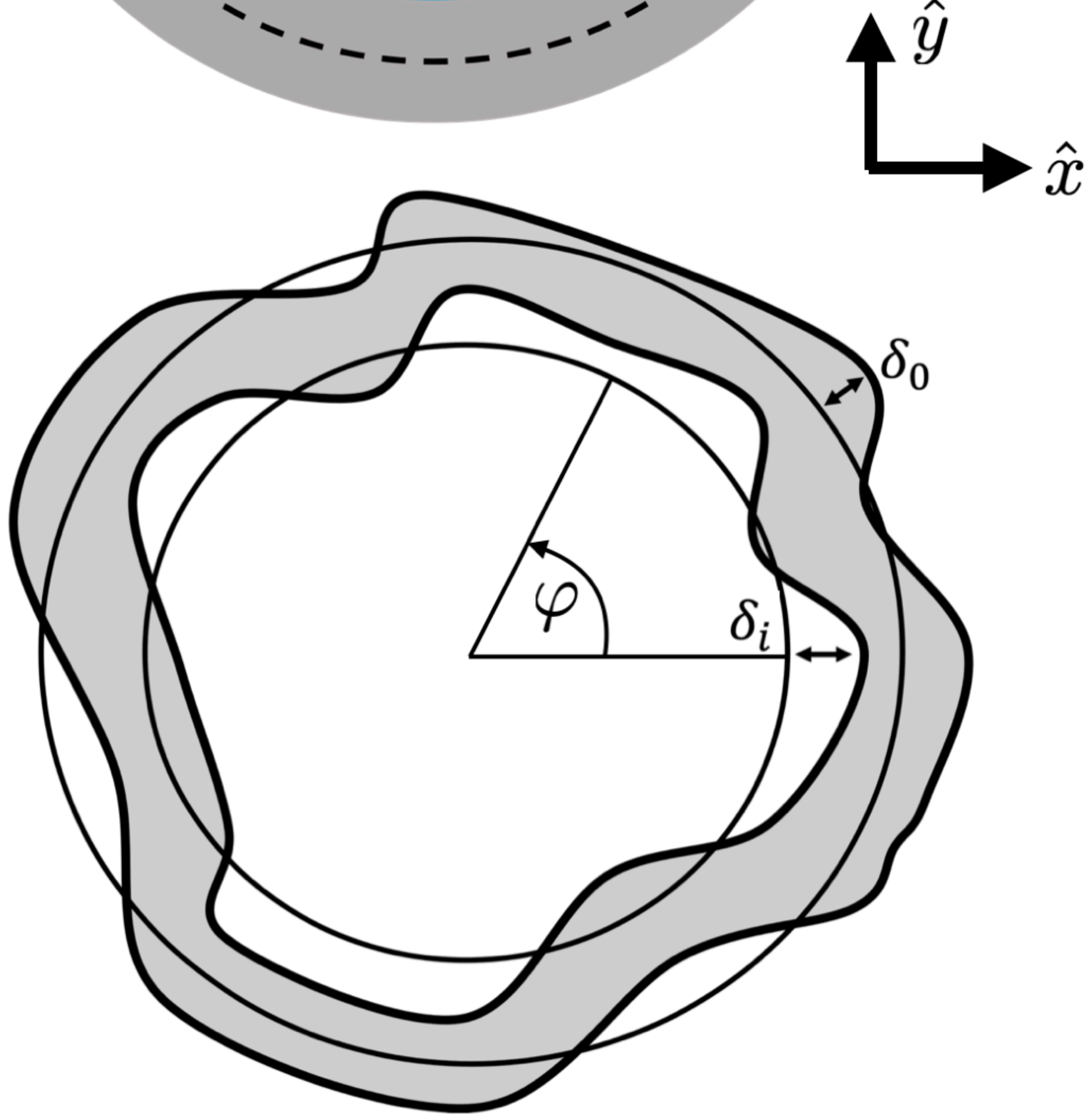
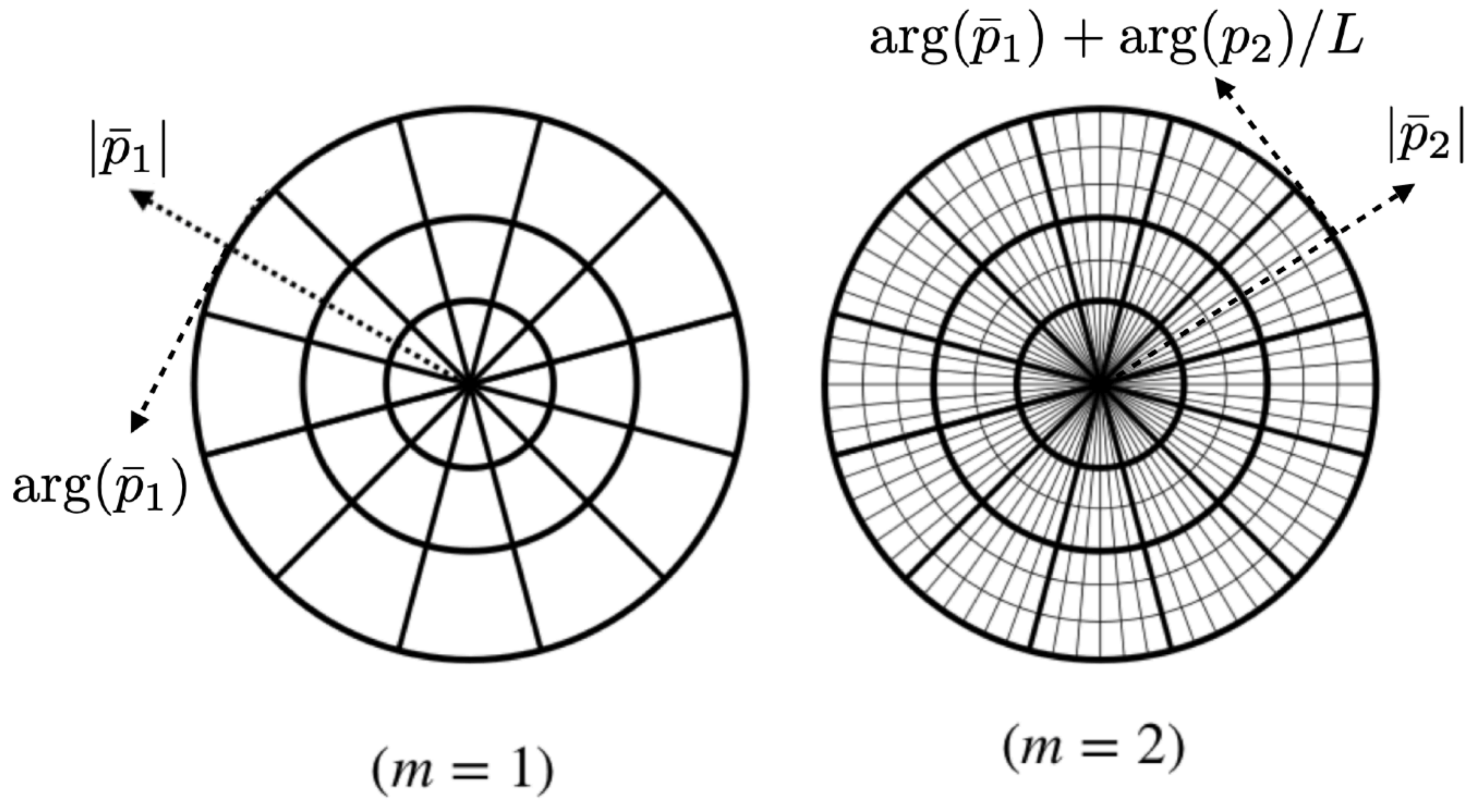
Parameters of model



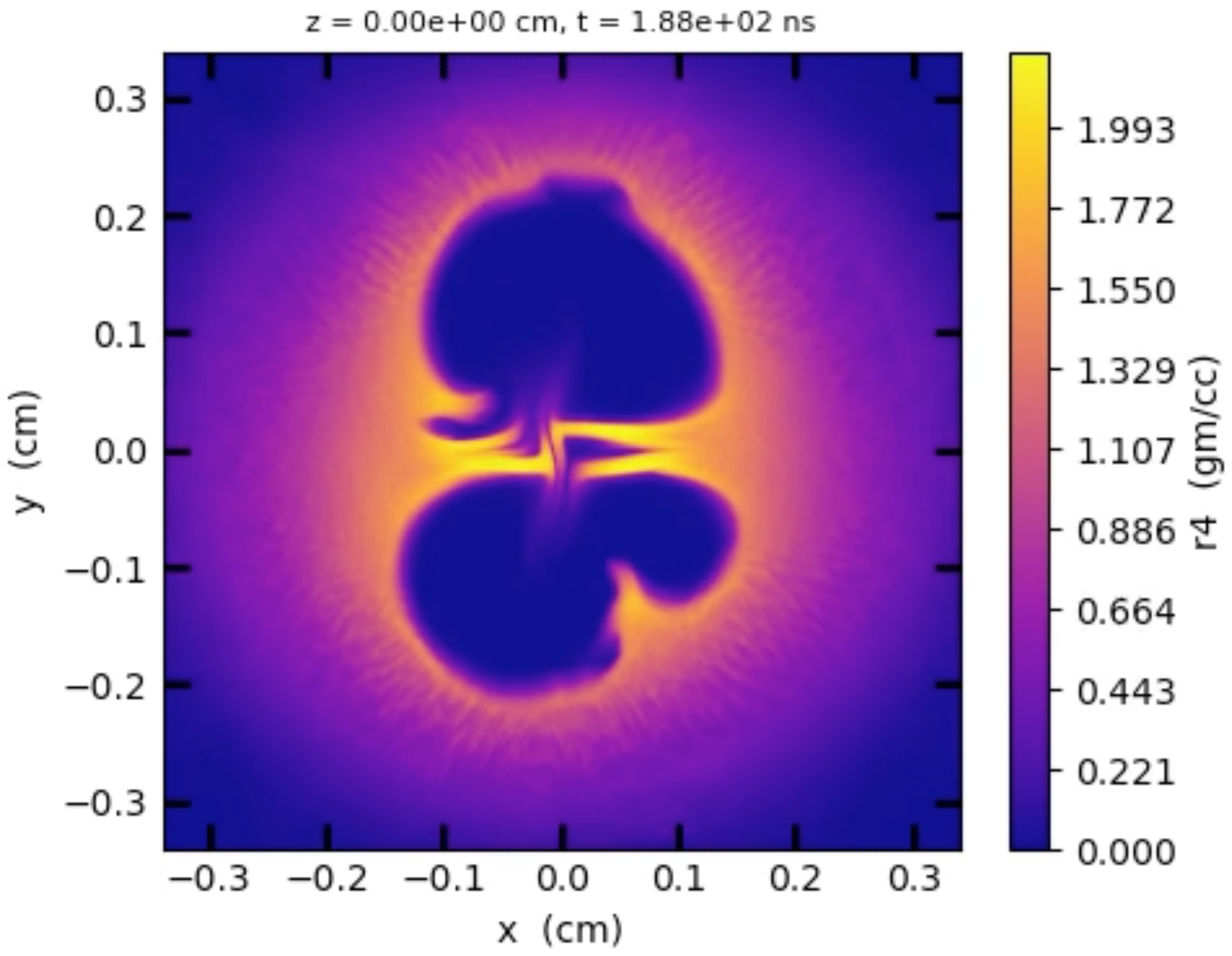


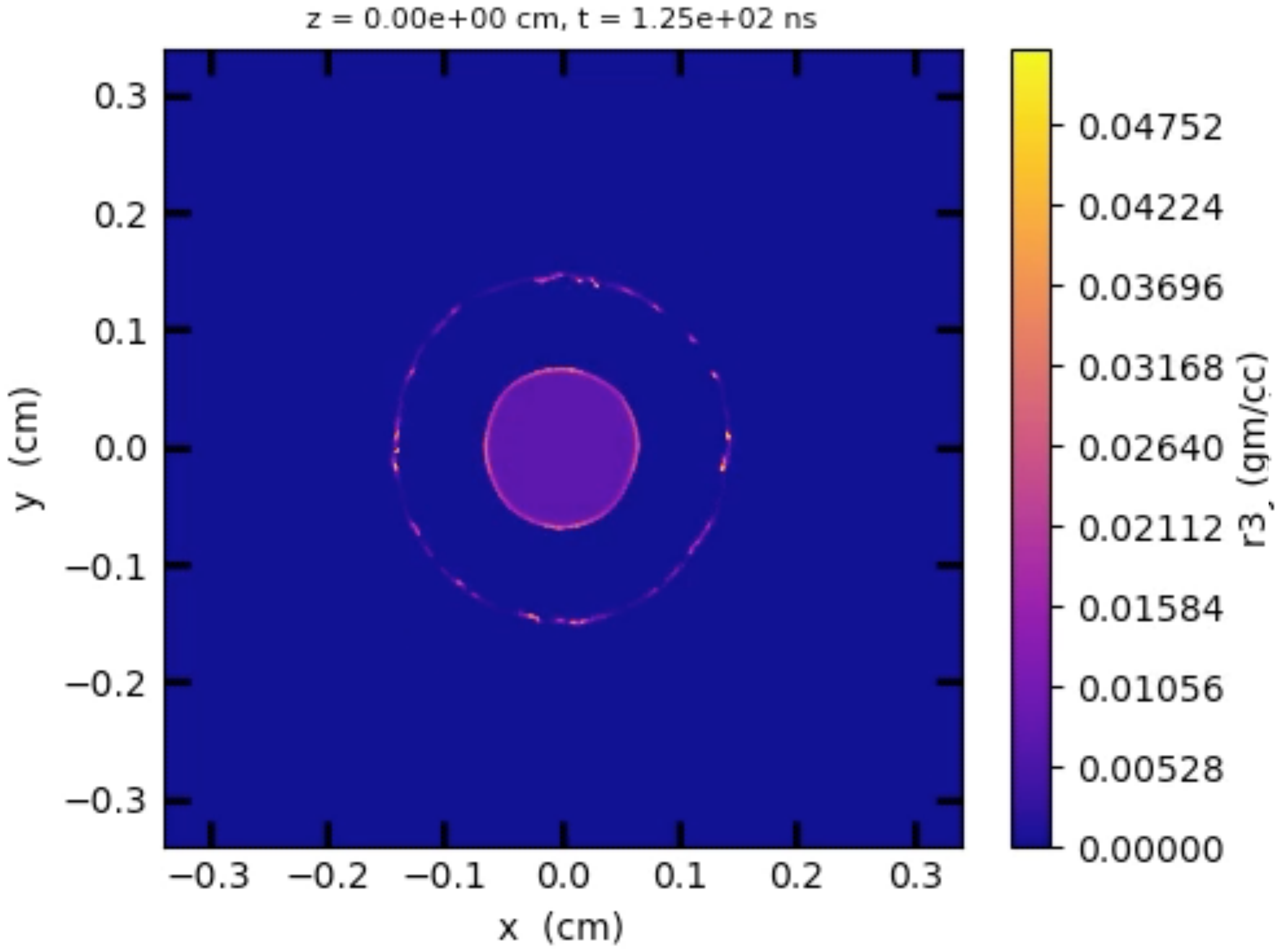


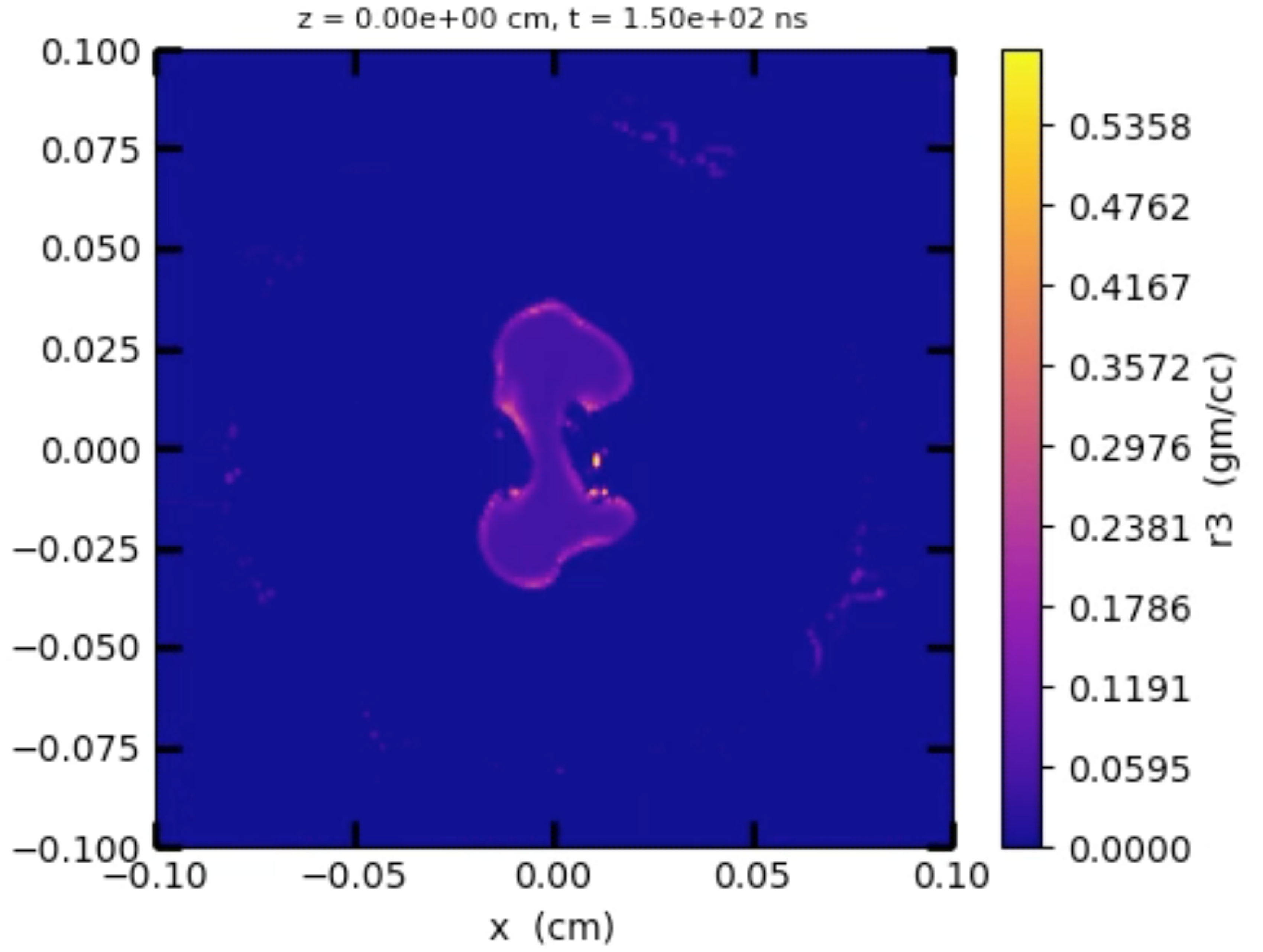


Example evolution

$$\mathrm{AR} = 3, T_o = 610\ \mathrm{eV}, \Delta = 1\%$$







Distribution description

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- control parameters
  - ✦ adiabat = preheat temperature = $\log_{10}(T_0)$ = [1.0, 2.8], T_0 = [10 eV, 630 eV]
  - ✦ liner aspect ratio = R_0/w = AR = [3, 9],   w = [800, 266] microns
  - ✦ time = [0.0, 200.0] ns (equally spaced by 2.5 ns from simulation)
- stochastic parameters
  - ✦ magnitude of liner perturbation (fraction of thickness, w) = $\log_{10}(\Delta)$ = [-2, -1], Δ = [1%, 10%]
  - ✦ phase of m=2 liner perturbation = ϕ_2 = [0, 2π]
  - ✦ phase of m=3 liner perturbation = ϕ_3 = [0, 2π]
  - ✦ phase of m=4 liner perturbation = ϕ_4 = [0, 2π]
- constant parameters
  - ✦ liner radius = R_0 = 2.4 mm
  - ✦ initial axial magnetic field = B_{z0} = 10 T
  - ✦ initial D_2 gas density = n_0 = 1 mg/cc
  - ✦ maximum axial current = I_{z0} = 10 MA
- dependent field parameter
  - ✦ liner density, $n_l(x, y; t)$ (resolution 10 microns)
  - ✦ magnetic field, $B(x, y; t)$
- sampling (539 samples x 81 times x 2 outputs = 87,318 256x256 images = 200k core*hrs)
  - ✦ 27 LHC samples in [$\log_{10}(T_0)$, AR, $\log_{10}(\Delta)$] and uniform in [ϕ_2, ϕ_3, ϕ_4]
  - ✦ 512 uniform in [$\log_{10}(T_0)$, AR, $\log_{10}(\Delta)$, ϕ_2, ϕ_3, ϕ_4]

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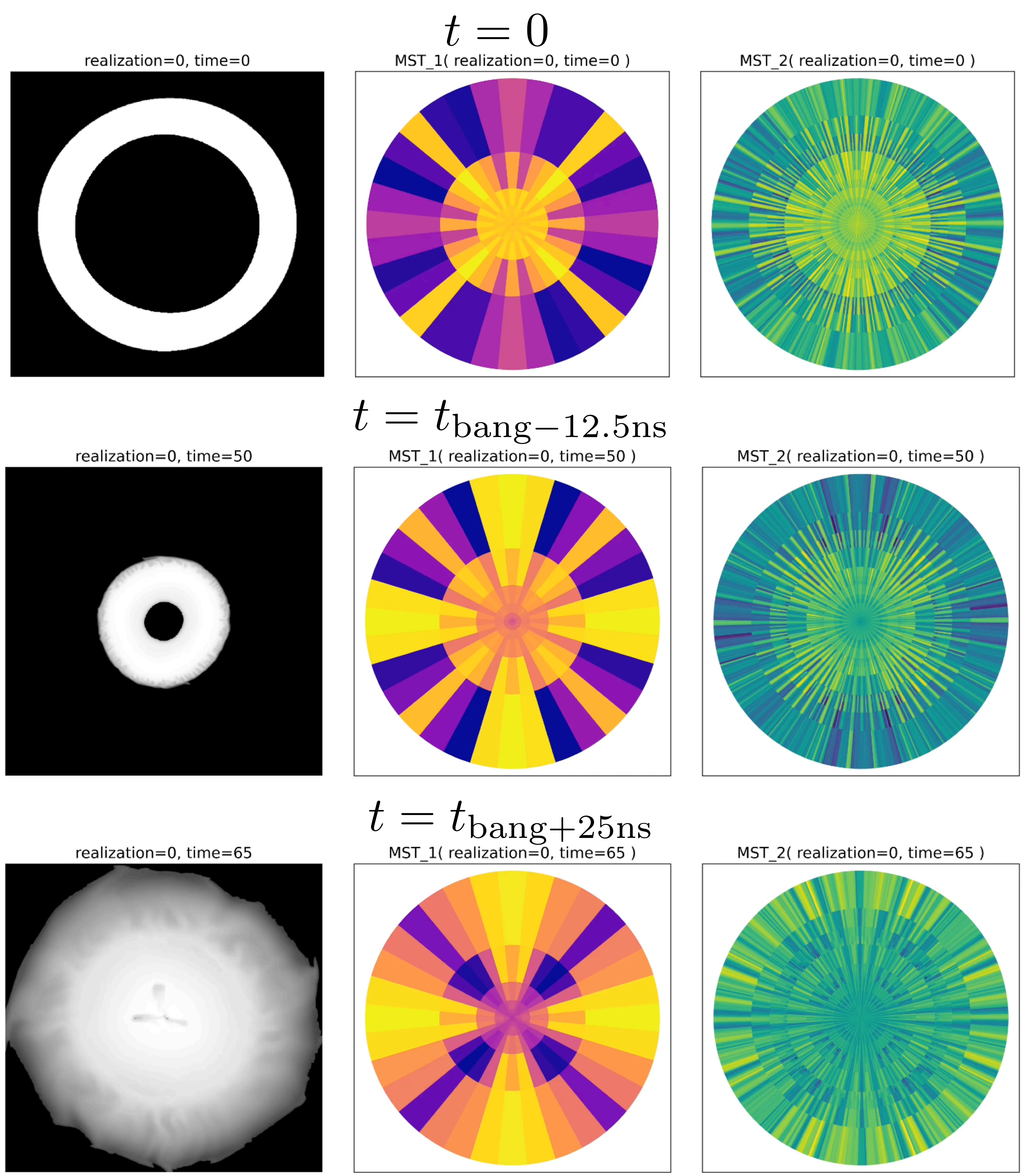
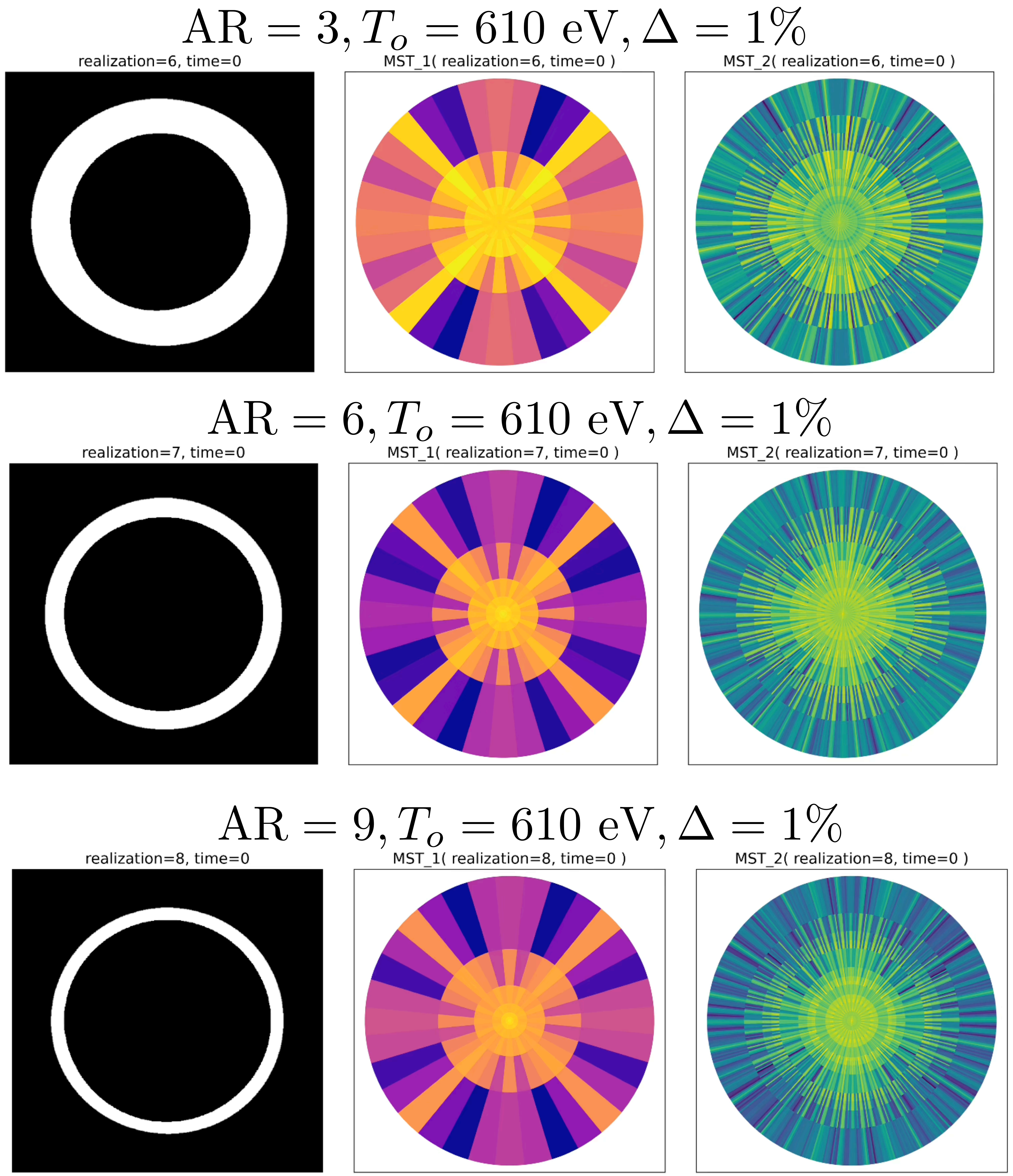
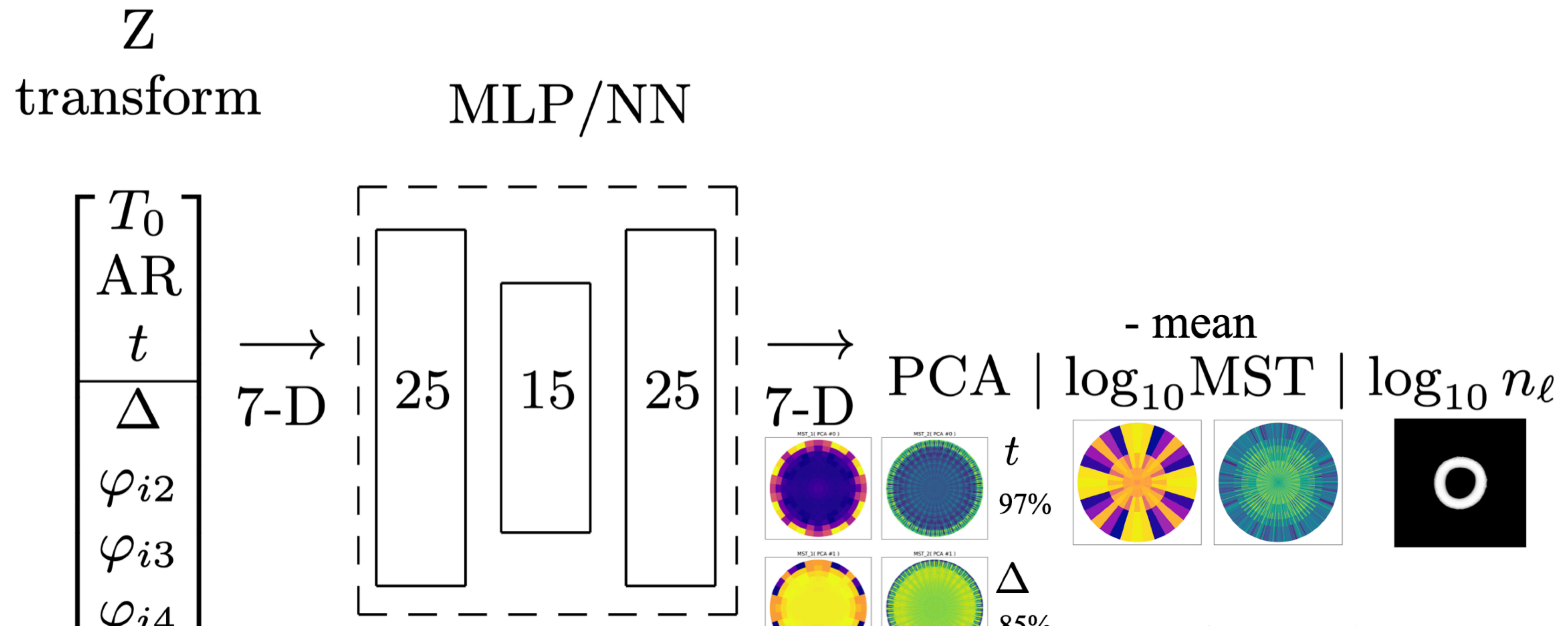
MST of liner density ensemble

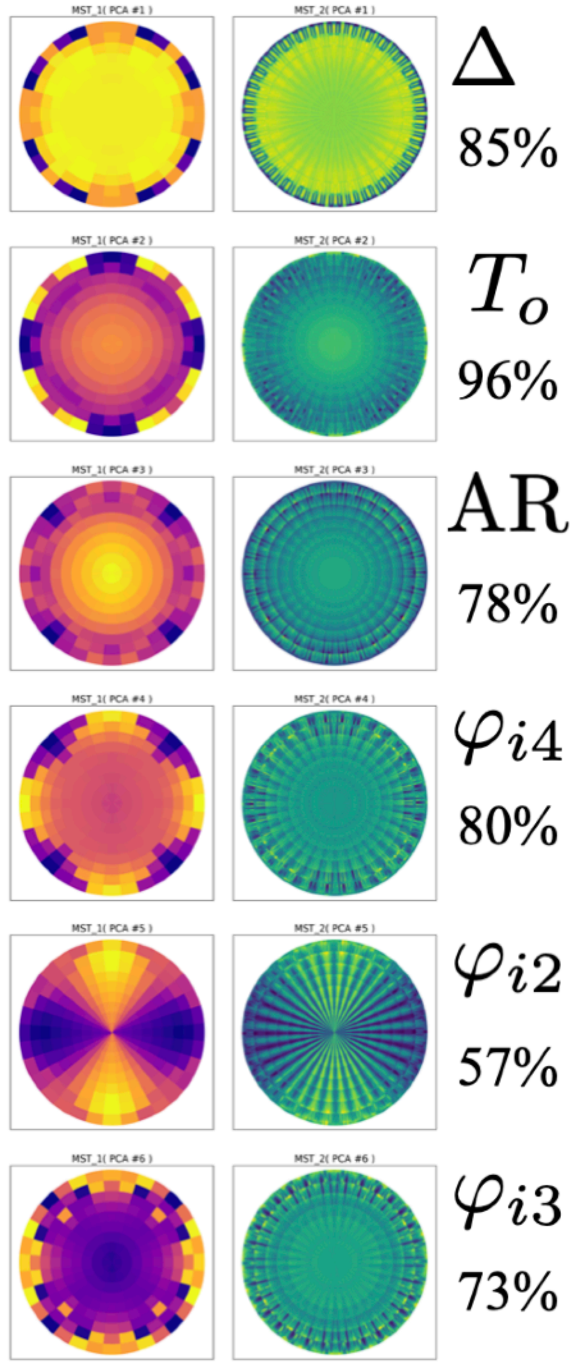


M=N=512
J=9
L=16



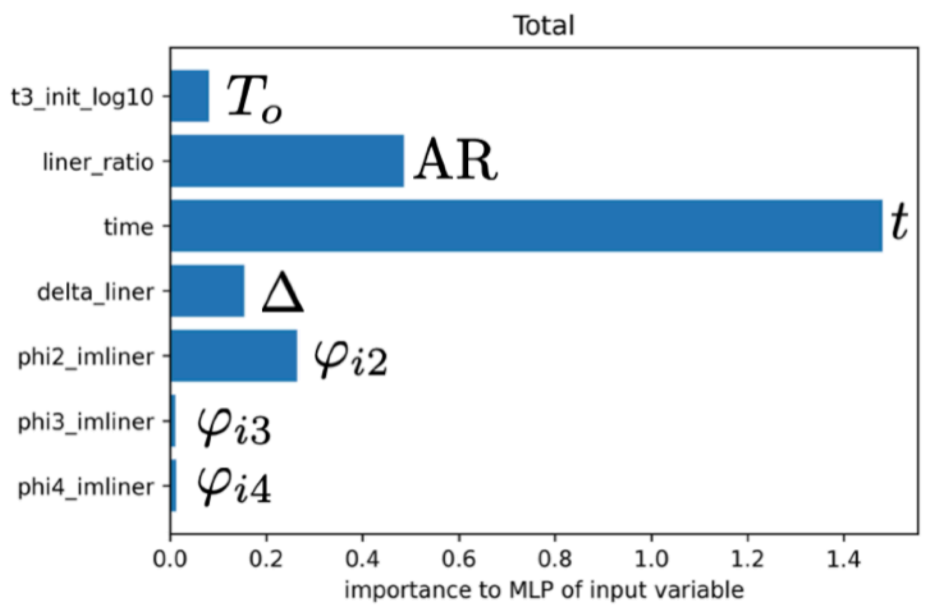
MLDL architecture

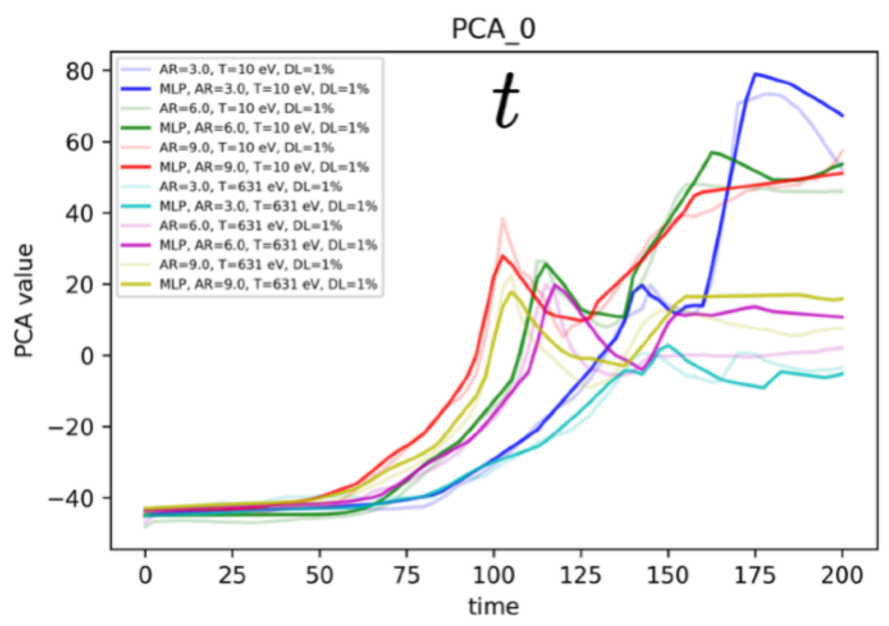




MLP/NN total score = 81%

PCA variance explained = 94%





Principal Component Analysis (PCA)

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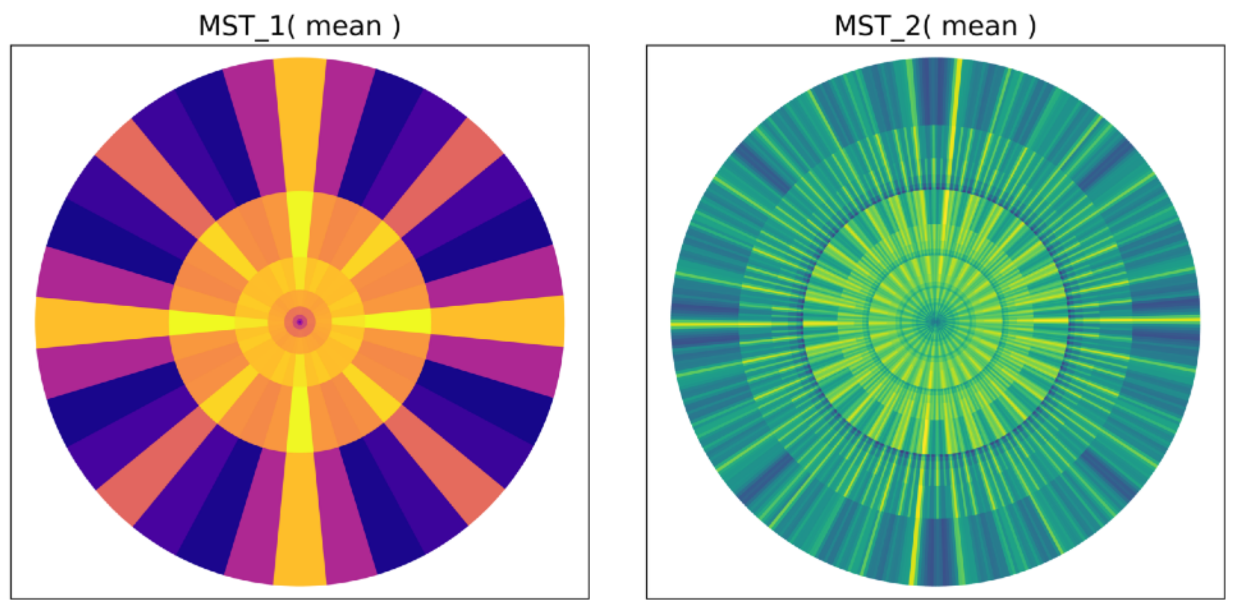

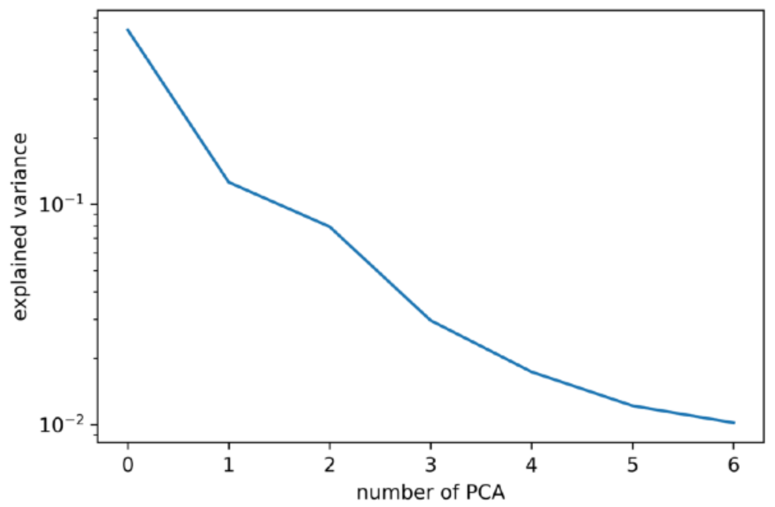

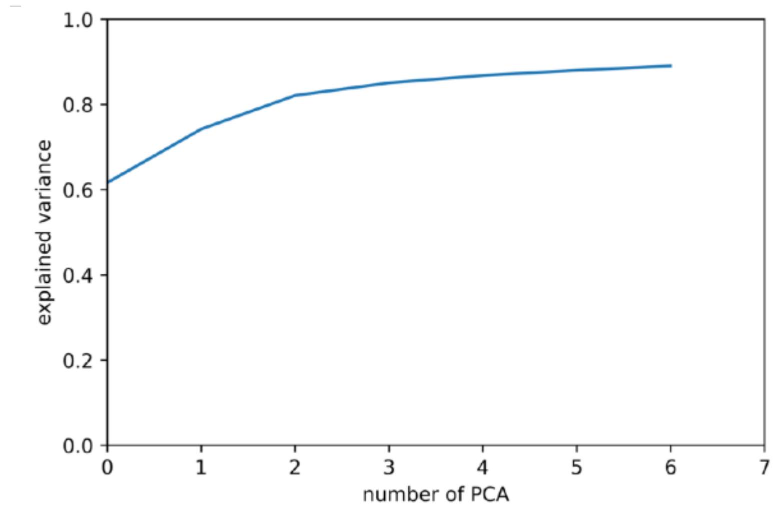

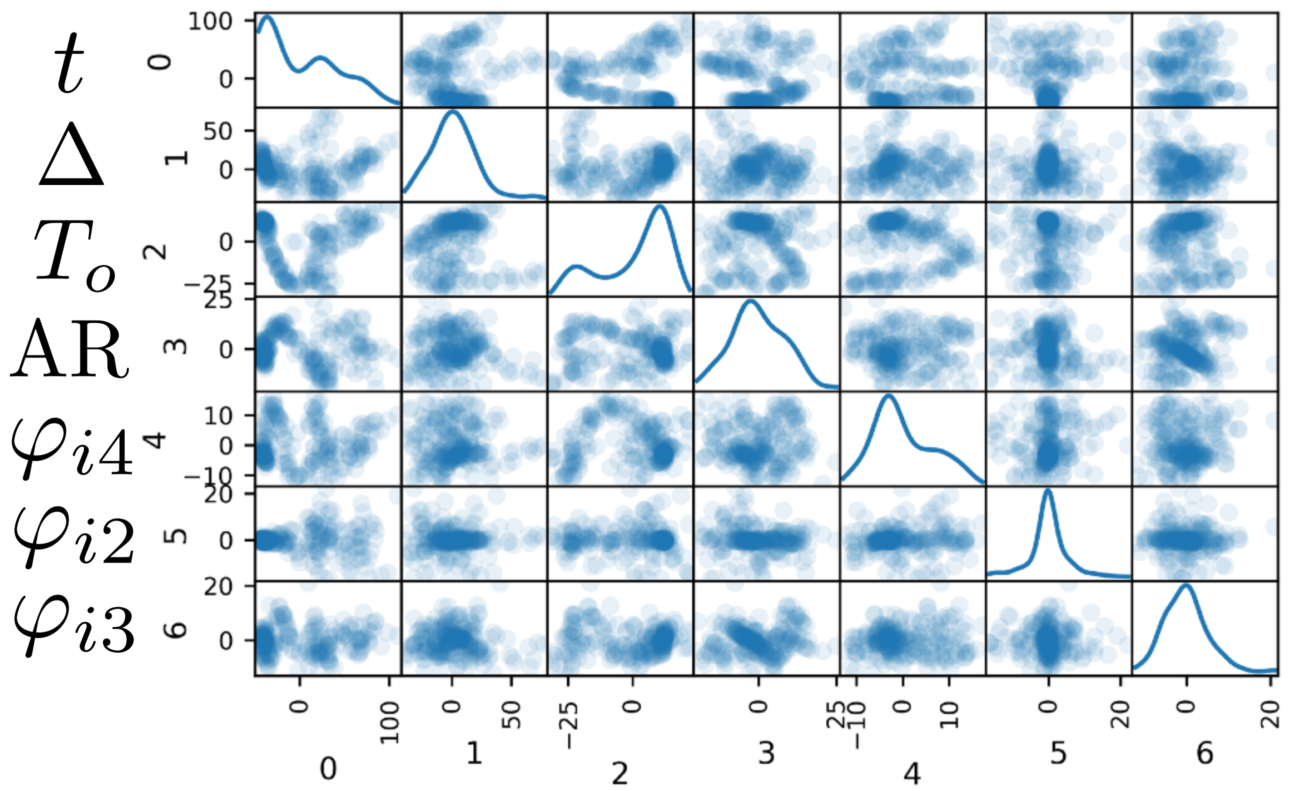

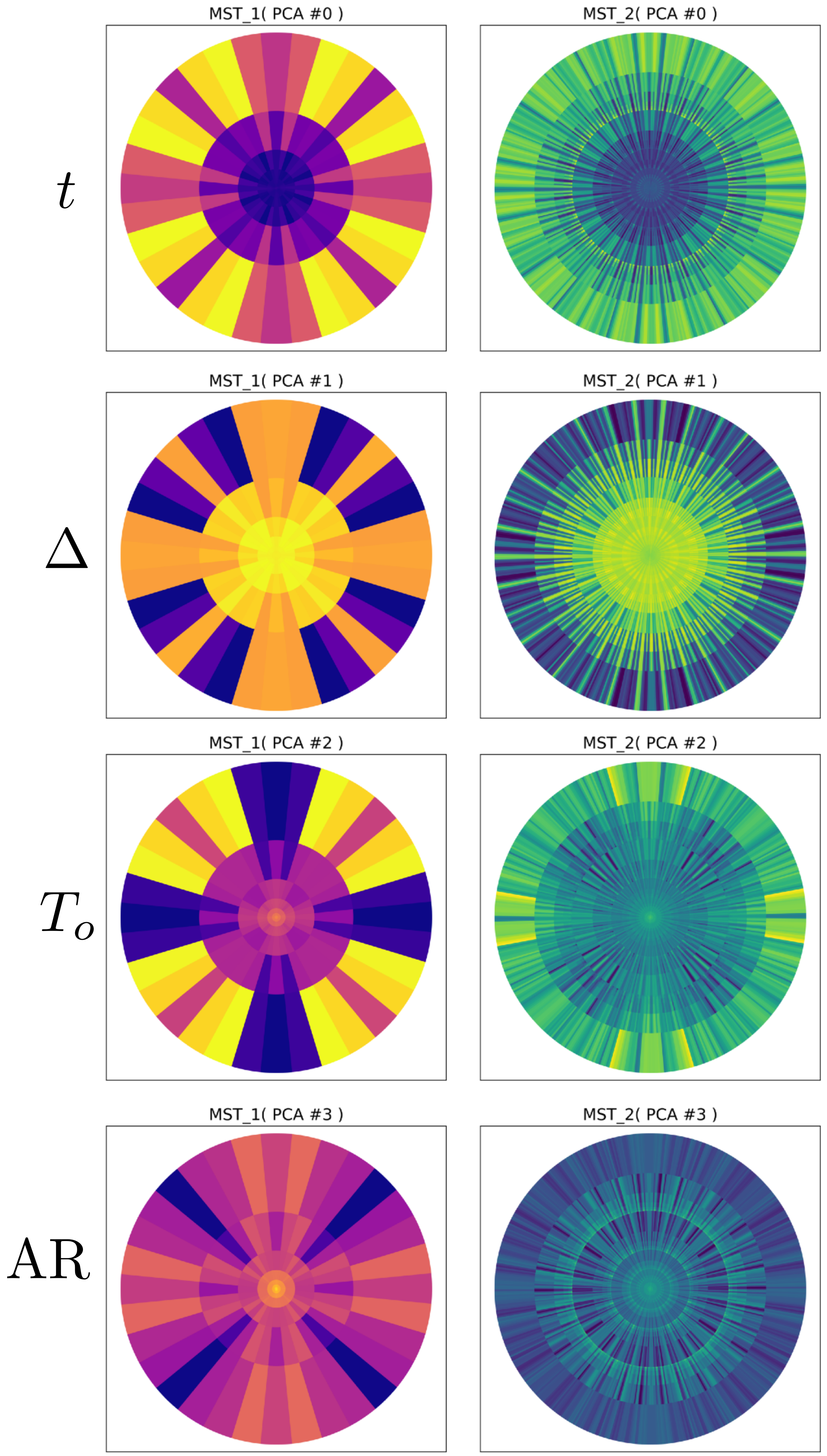

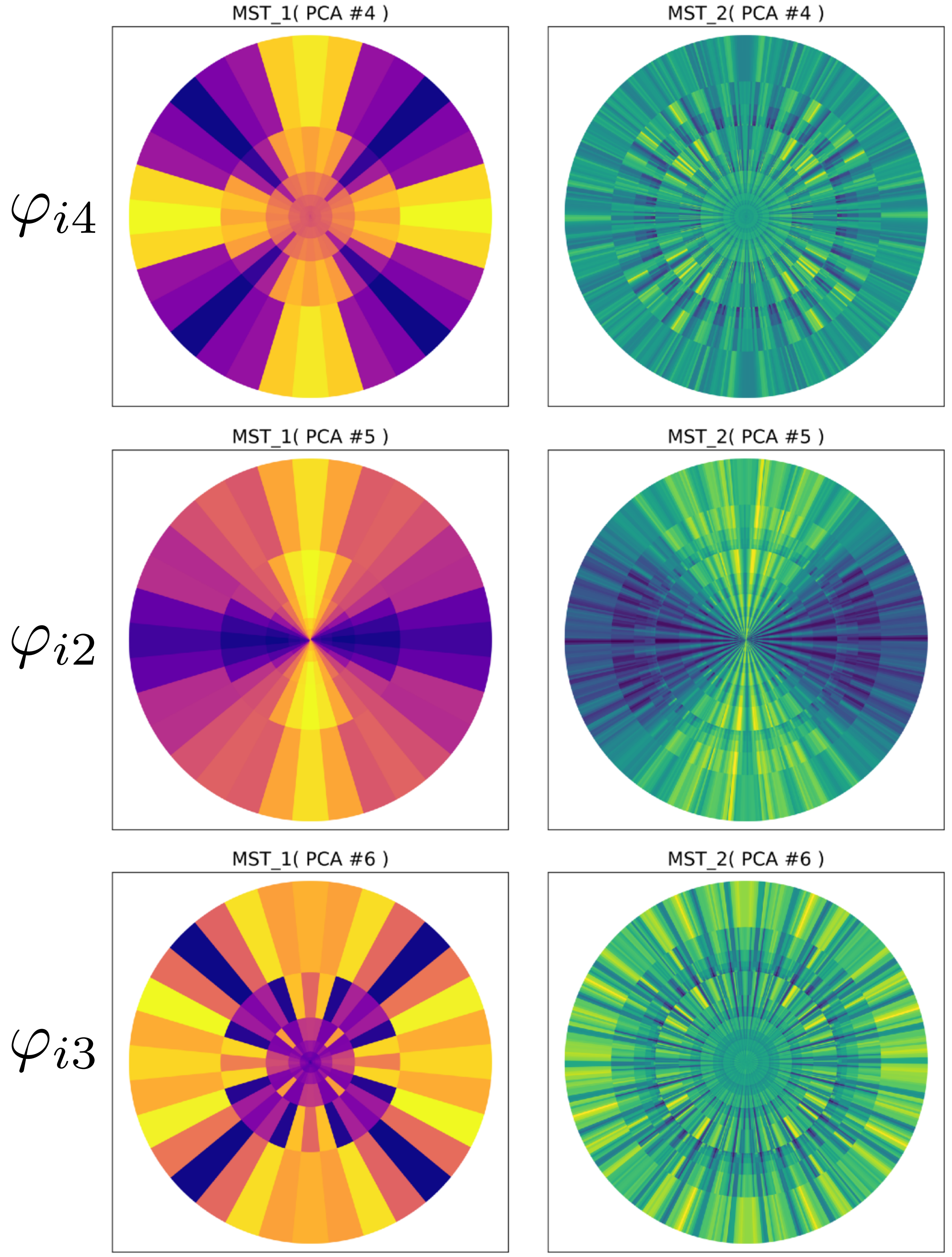

Singular Value Decomposition (SVD) of cross correlation of input parameters to Mallat Scattering Transformation

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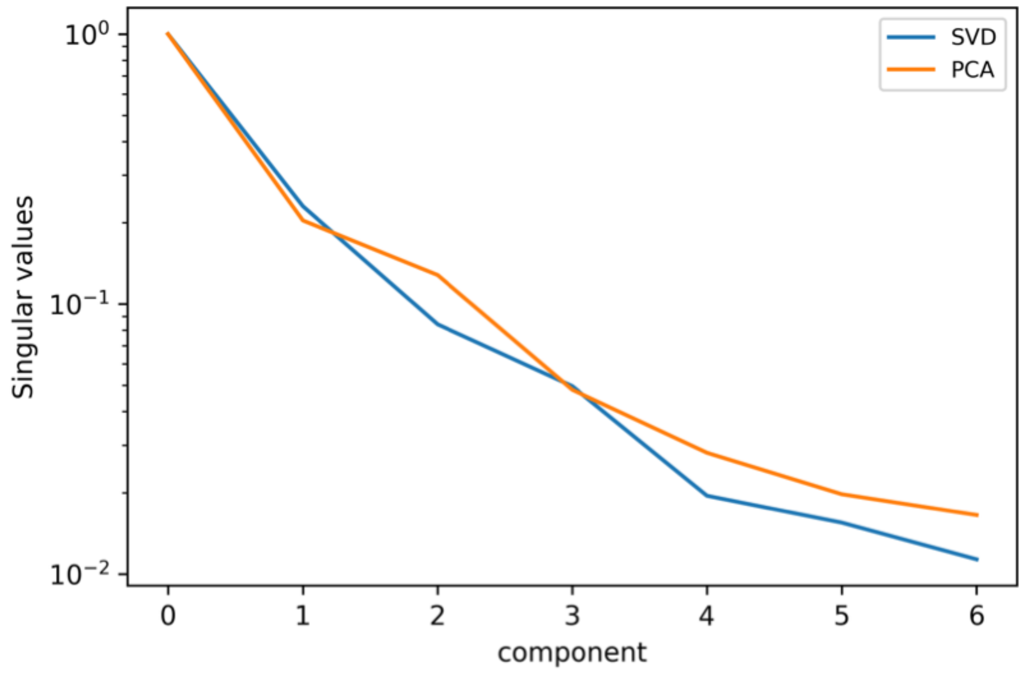
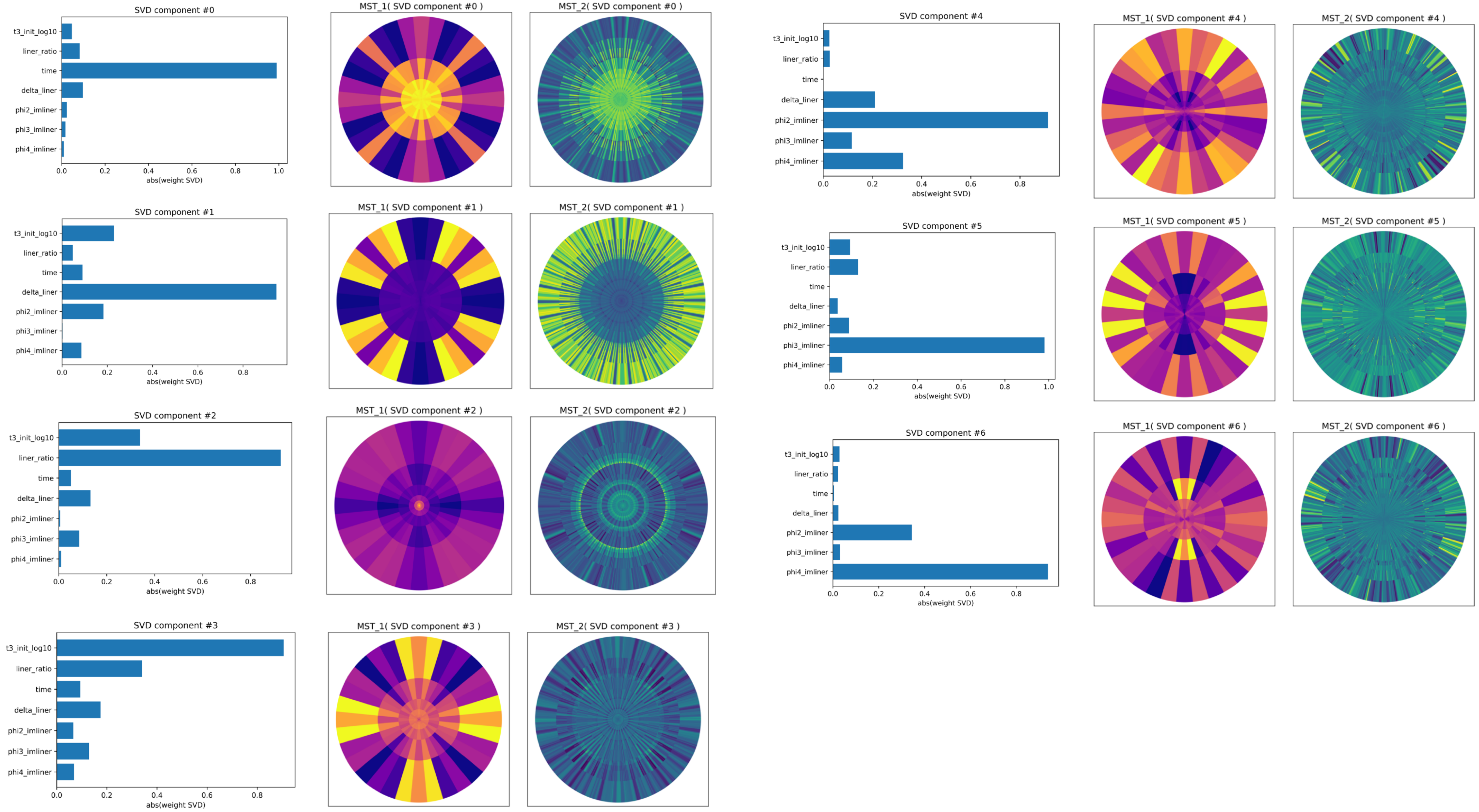
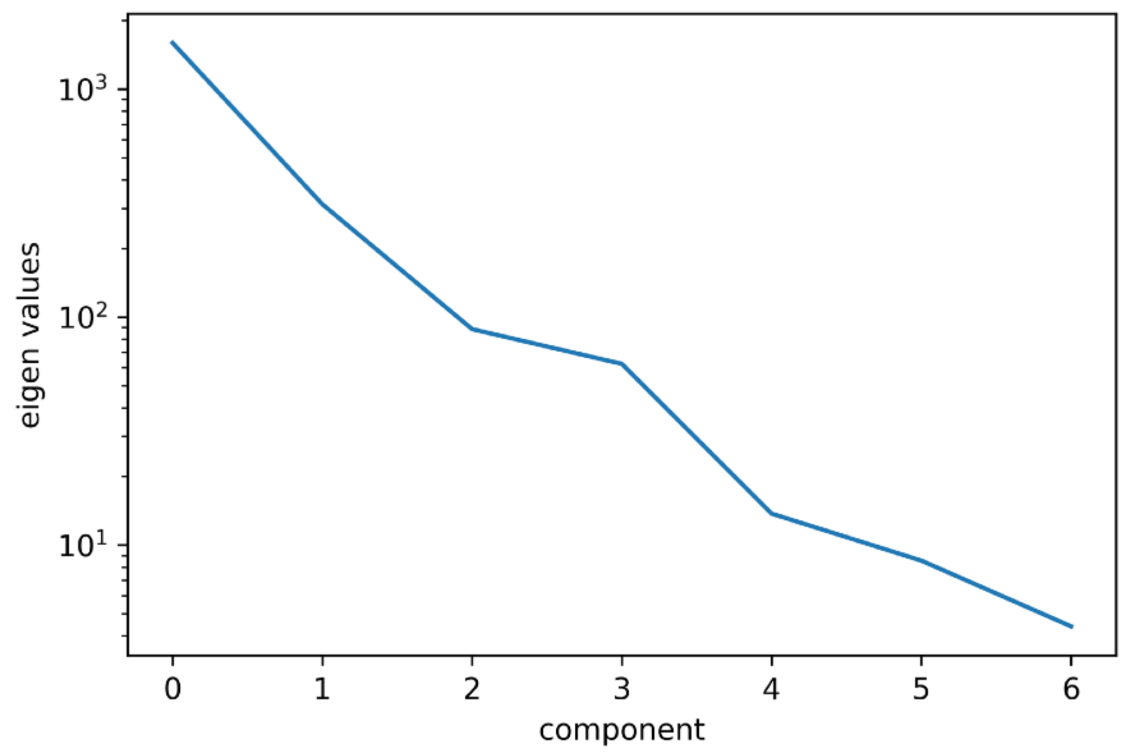
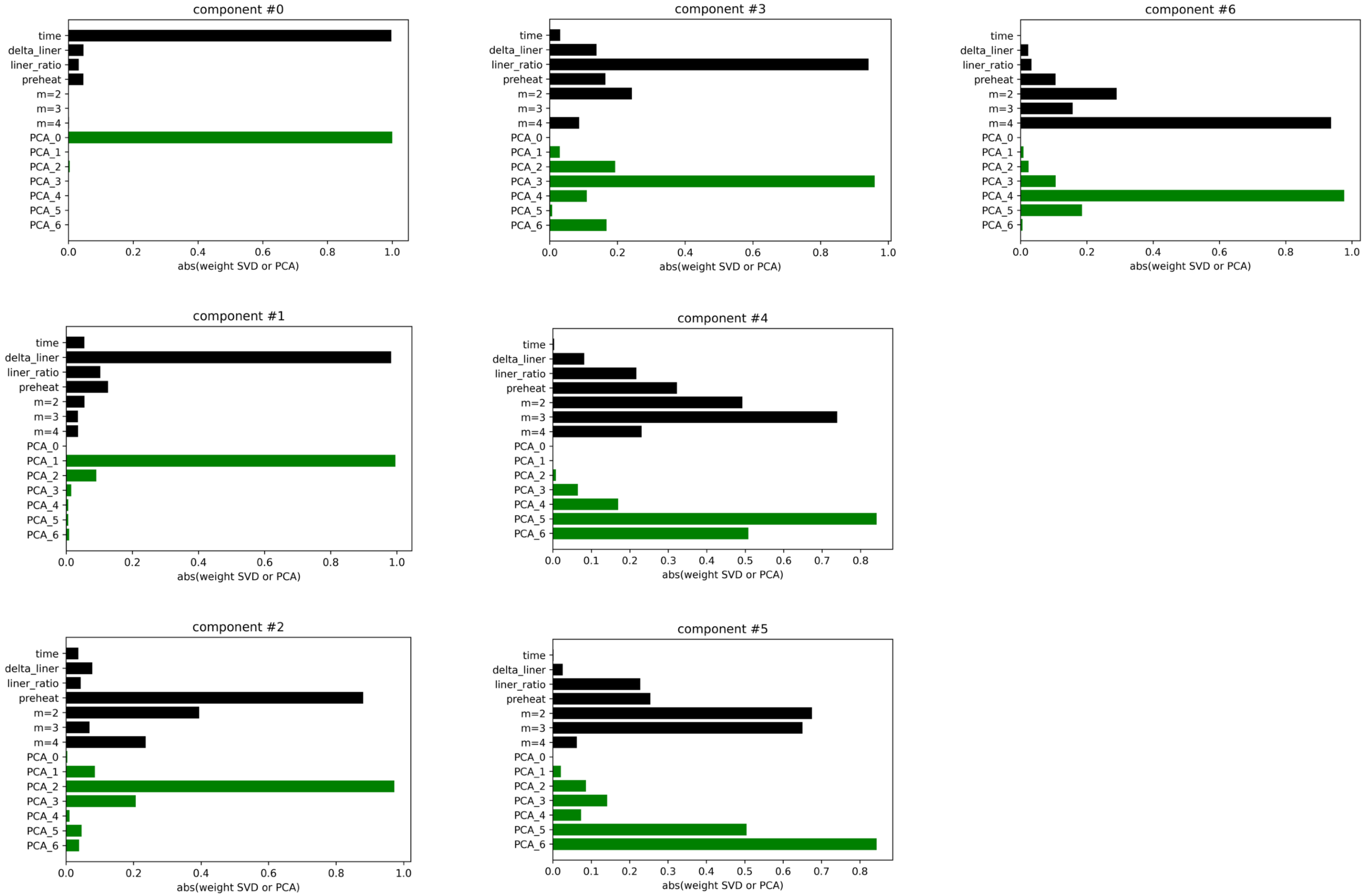




Singular Value Decomposition of PCA to SVD





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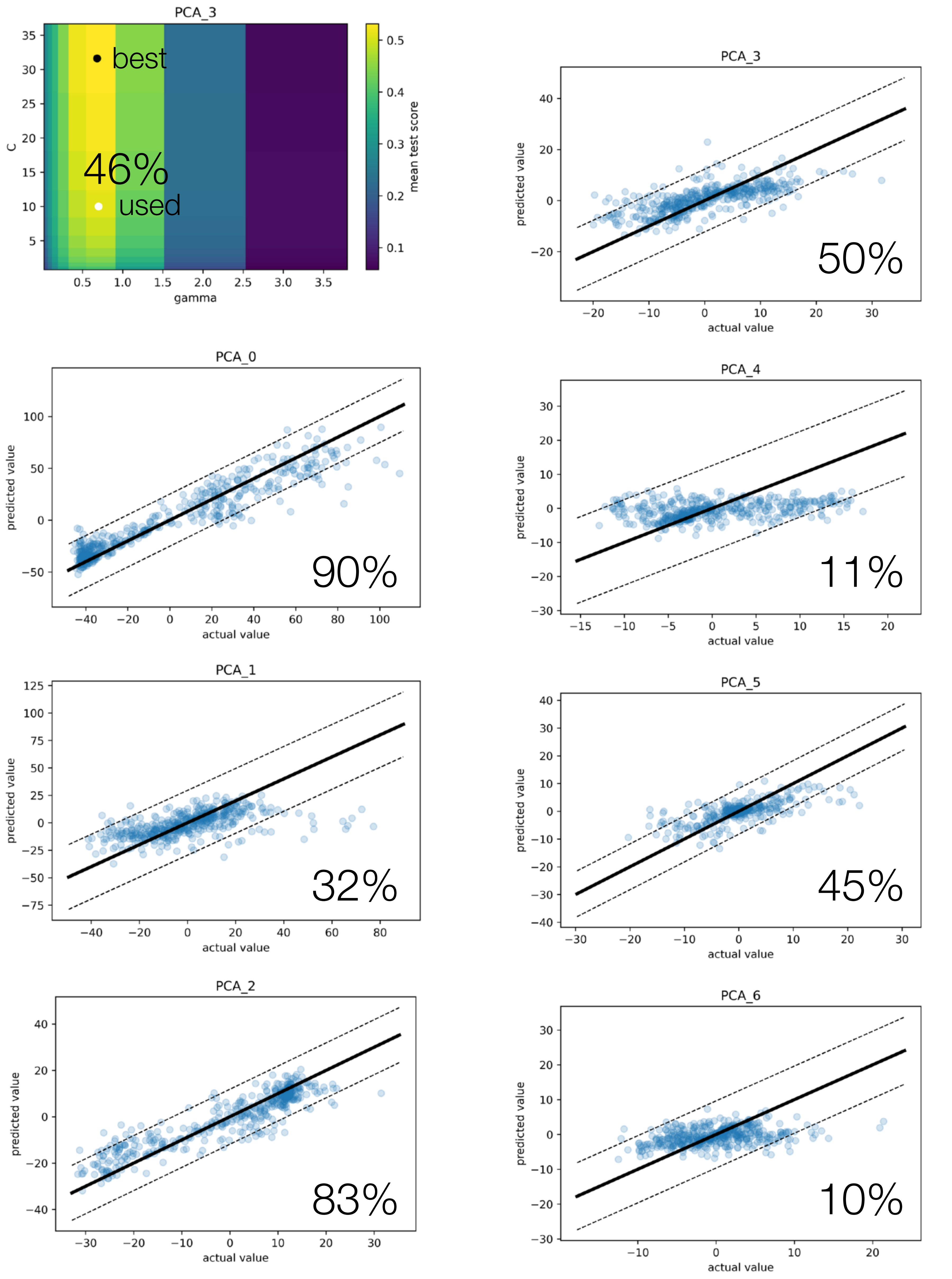
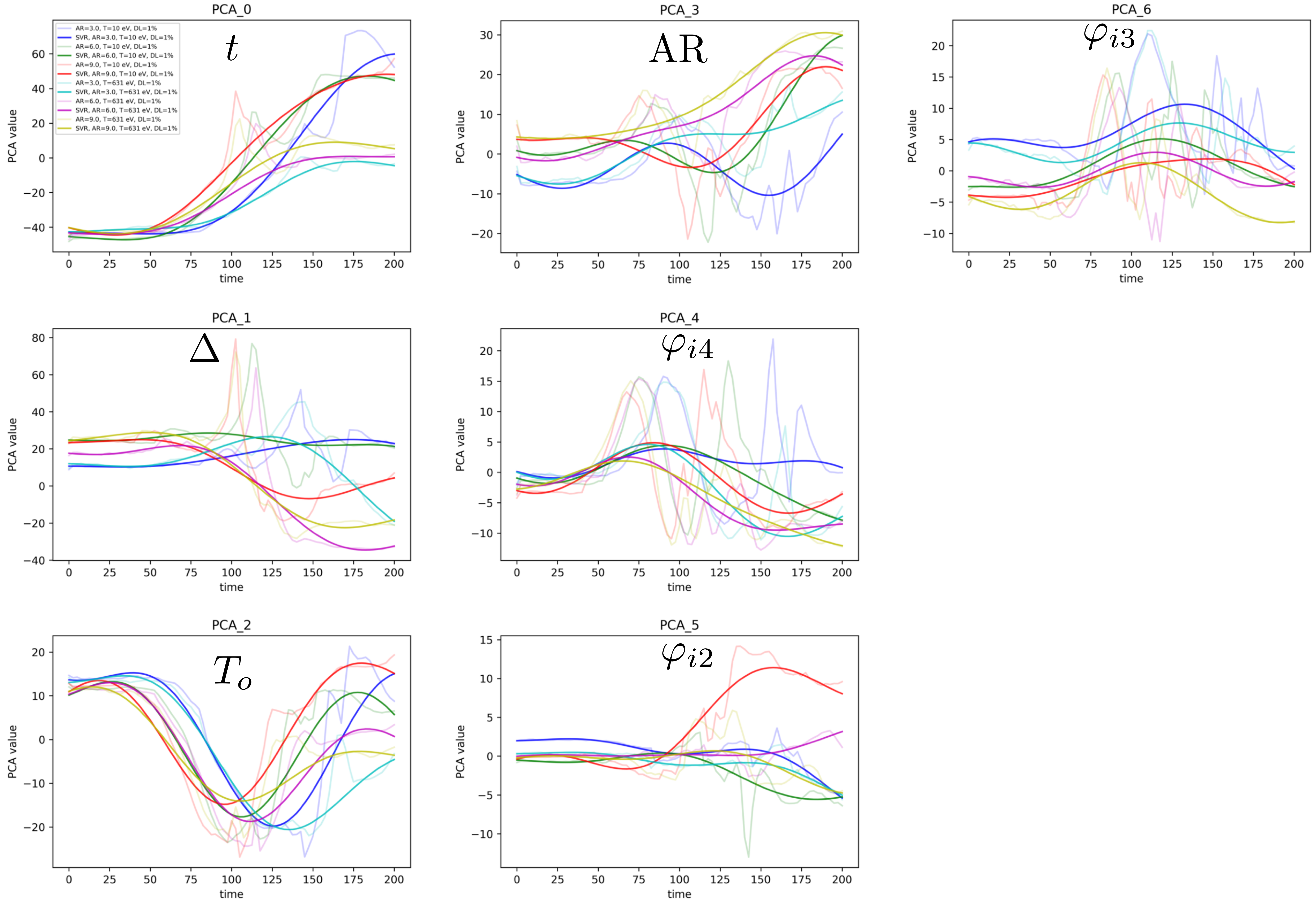
Support Vector Regression (SVR)





Neural Network / Multi Layer Perceptron (NN/MLP) regression

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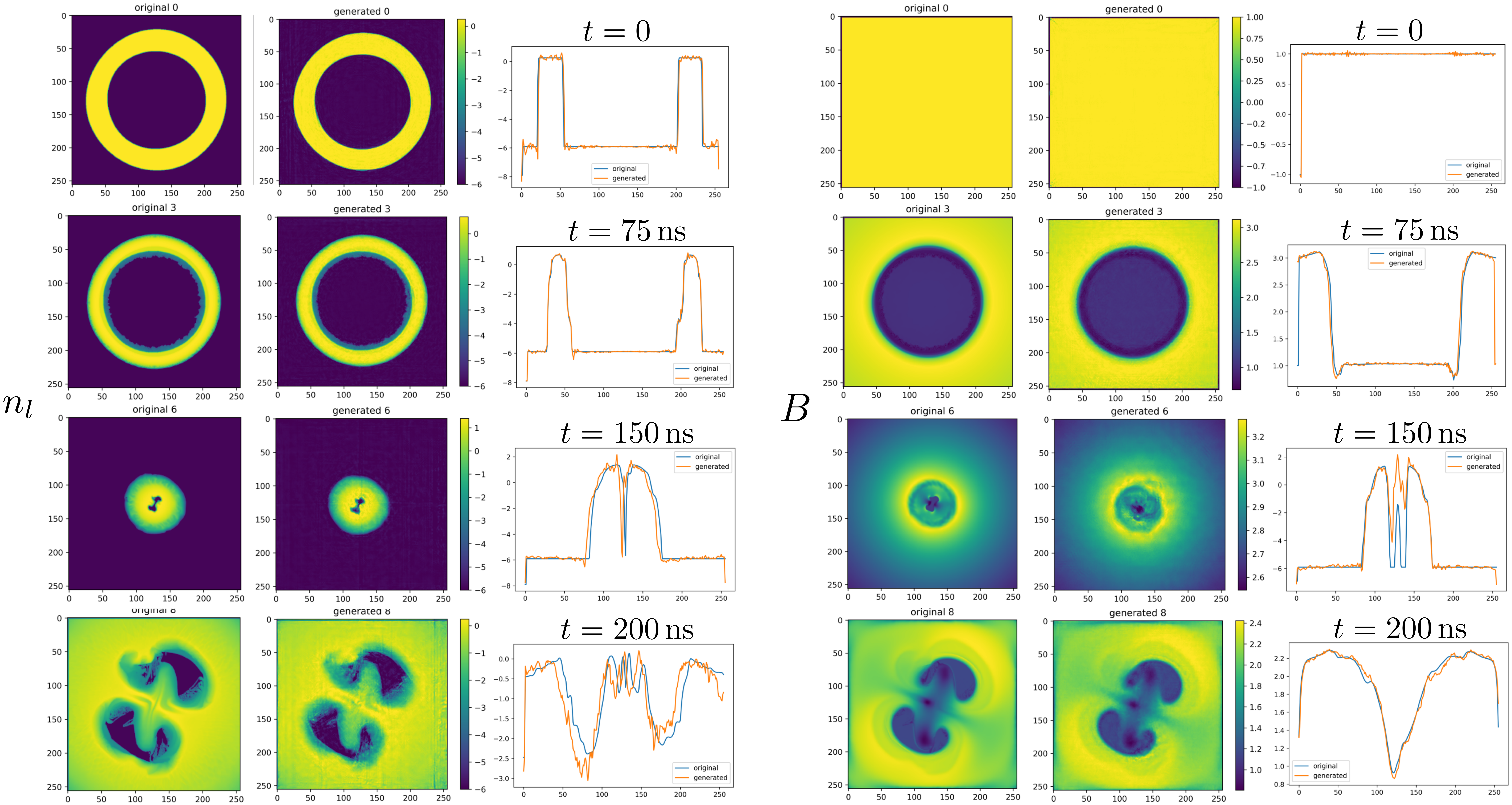
Results of Wavelet Phase Harmonics (WPH) analysis



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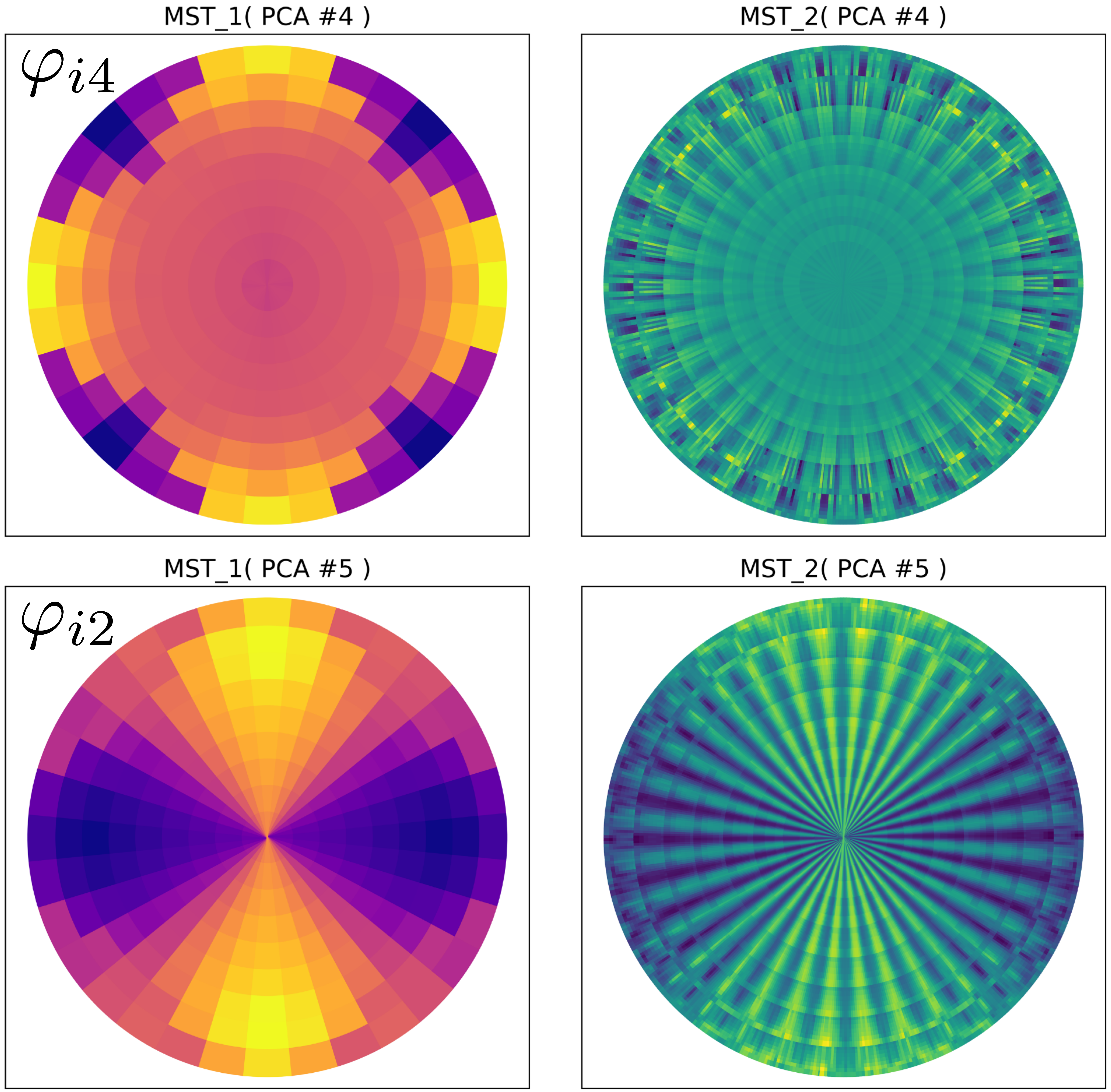
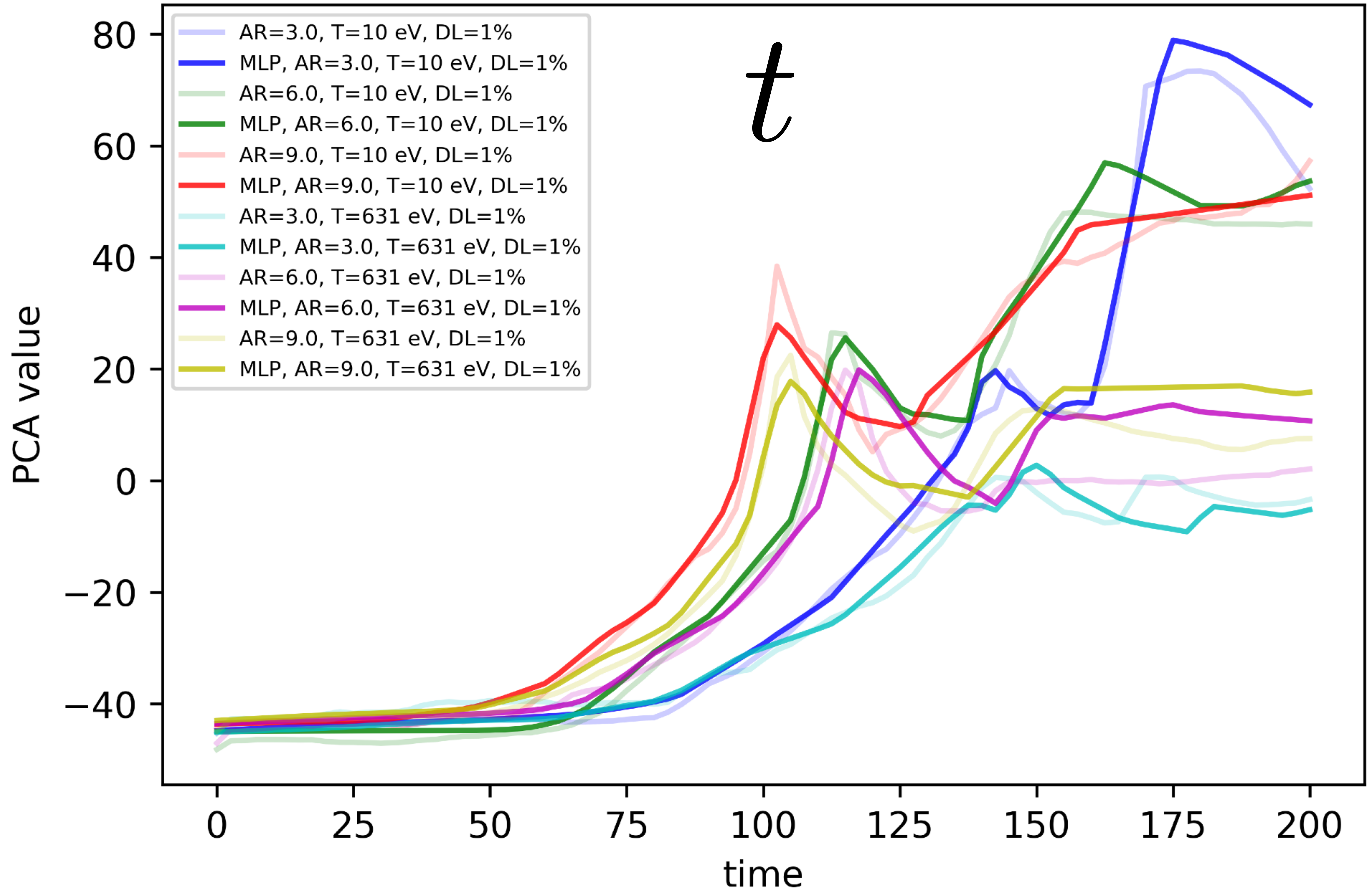
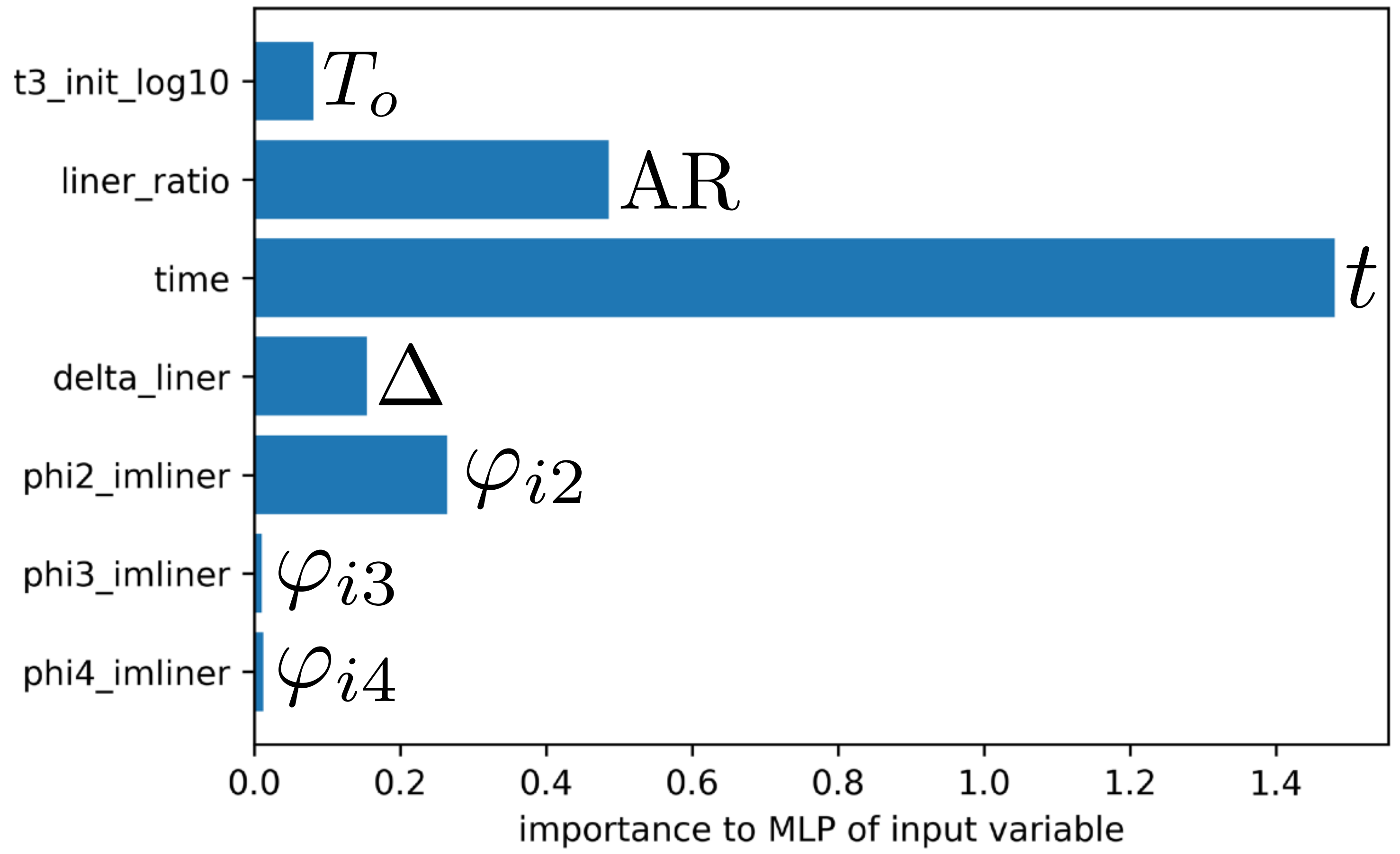
A closer look at the results

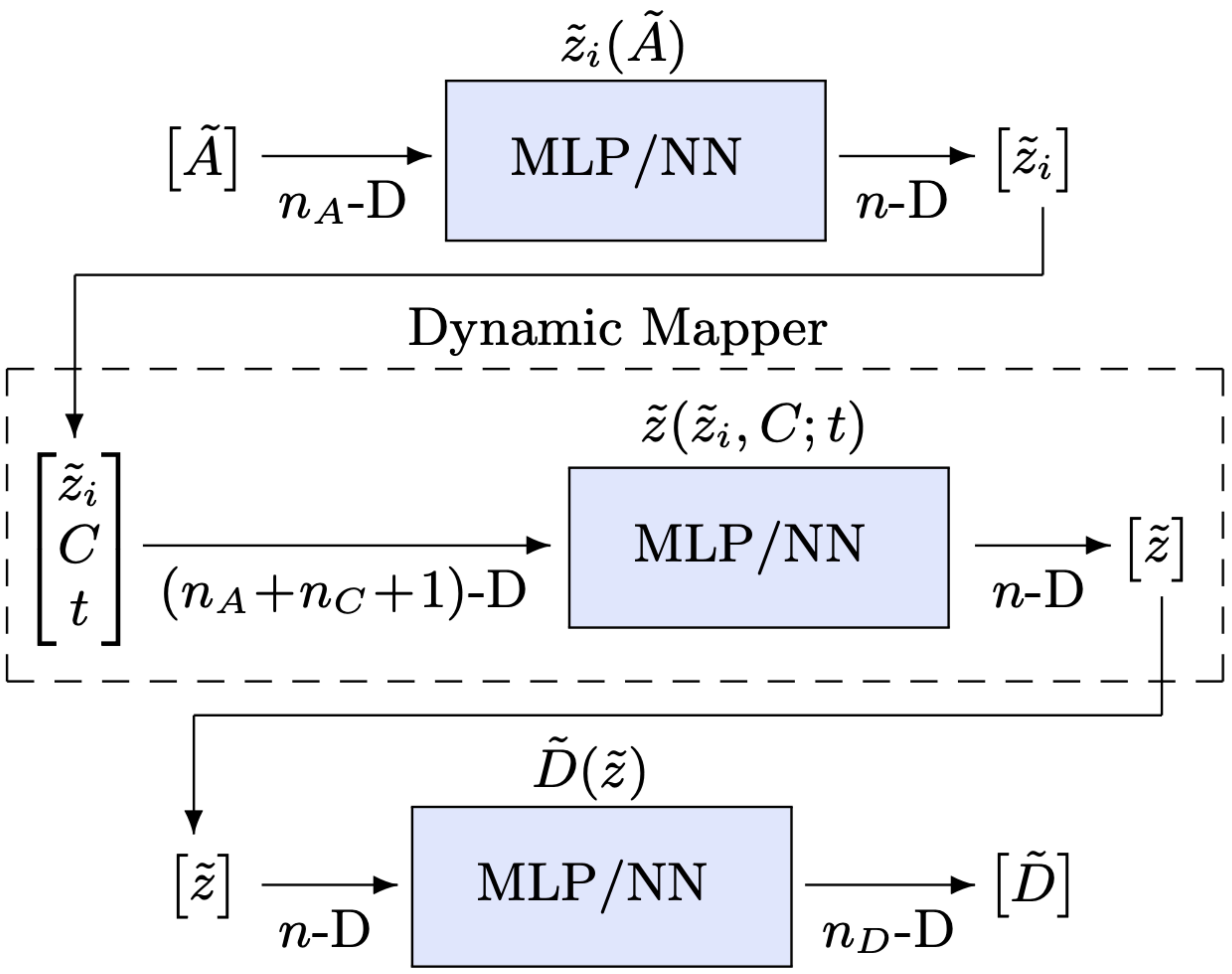




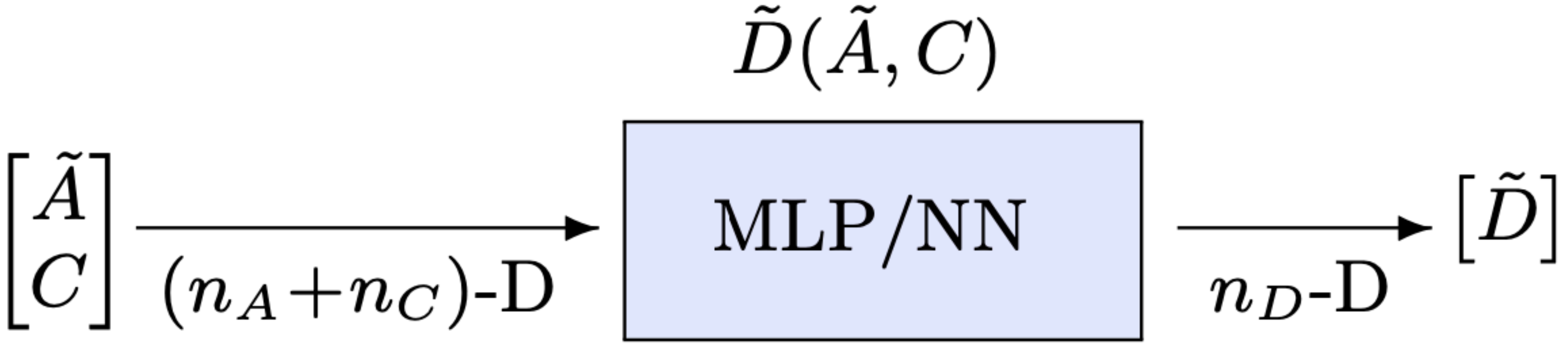


Variations on MLDL architecture

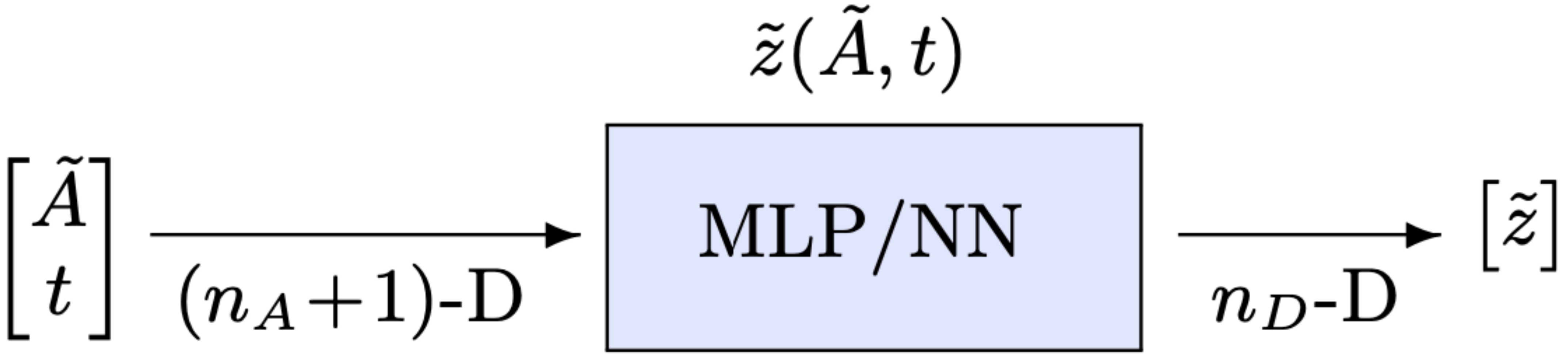
Decomposed approach



Composite approach



Current work



The geometry of the workflow — Heisenberg Scattering Transformation (HST)

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$$\mathrm{i}\, H_m[f(x)](z) = \phi_{px} \star \left(\prod_{k=1}^{m} \mathrm{i} \ln R_0 \psi_{p_k} \star \right) \mathrm{i} \ln R_0 f(x)$$

Logarithmic Generating functional (LG) of $H(z)$

$F[f, \pi]$ → HST → $|\beta(z)\rangle$ → PCA → $|\beta_i(z)\rangle$ → MLP → $H(\beta)$ or $|\beta_{0k}\rangle$ and $H \otimes dH$

$F[f, \pi]$: Banach space of all functionals

$|\beta(z)\rangle$: $\dim(|\beta(z)\rangle) = N^{2n_x}$

Hilbert space of all renormalization functions

PCA: $\mathrm{LG}''(z) = 0$

$|\beta_i(z)\rangle$: low dimensional complex linear subspace $\mathbb{C}^n$

dynamical manifold or solutions of Renormalization Group Equations (RGE)

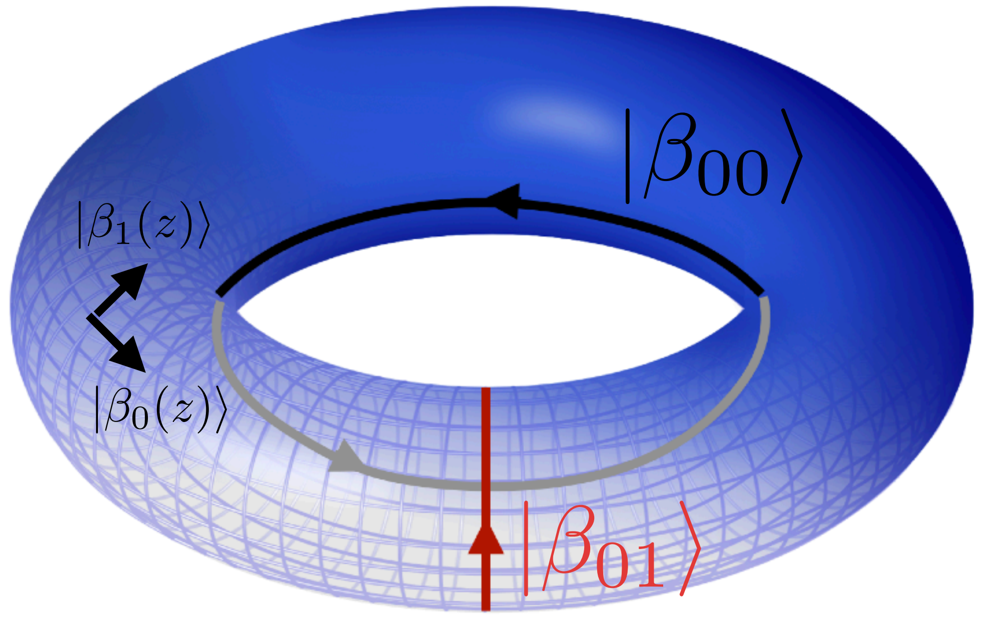
$H(\beta)$ or $|\beta_{0k}\rangle$ and $H \otimes dH$:

$\mathrm{Re}(H) = \text{constant} \Rrightarrow \text{H group action}$
$\mathrm{Im}(H) = \text{constant} \Rrightarrow \text{dH group action}$

$\dim(|\beta_{0k}\rangle) = 2n_g < 2n \ll N^{2n_x}$

<u>topology</u>
analytic function, $H(\beta)$
or
ground states (topological obstructions or homology classes)
+ foliations (H and Ad(H) leaves)

$z \equiv p + \mathrm{i}\, x$
$f(x) \equiv$ field or probability distribution
$\pi(x) \equiv$ conjugate field momentum
$n_x \equiv$ dimension of x or the base manifold
$N \equiv$ number of grid cells of x
$n \equiv$ dimension of f or the base dynamical manifold, or number of fields
$n_g \equiv$ number of ground states



Topological discovery of dynamical systems

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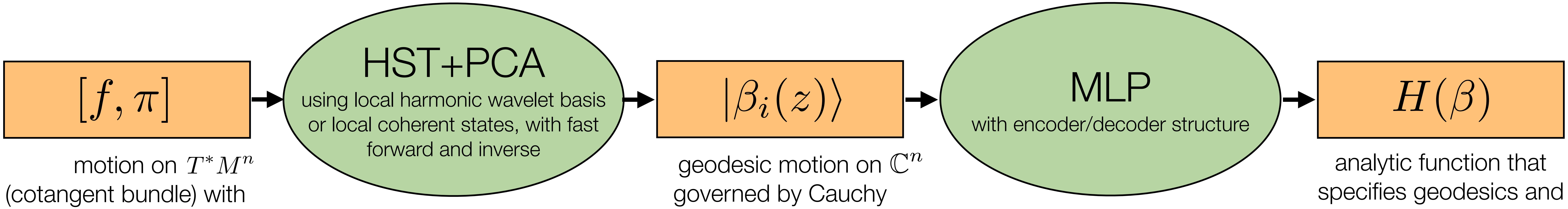

motion on T^*M^n (cotangent bundle) with symplectic metric $\mathrm{d}f$^$\mathrm{d}\pi$ and H[f,π] governed by Hamilton's equations

geodesic motion on $\mathbb{C}^n$ governed by Cauchy Riemann equations, equivalently the dimension and basis vectors (solutions to the Renormalization Group Equations) of the dynamical manifold

analytic function that specifies geodesics and adjoints, equivalently the topology of the dynamical manifold

note: the **S-matrix** or complex curvature is

$$\frac{d^m H(\beta)}{d\beta^m}$$

Discussion and conclusions

- fast, high fidelity, surrogate developed for resistive MHD
  - 10^7 acceleration over conventional finite volume calculation (from 360 core*hrs to less than 0.1 core*sec)
  - simple MLDL architecture that is fast to train (1 core*sec PCA, 20 core*sec MLP + 16/27 GPU*hrs for MST/WPH on training data, 200k core*hrs to generate training data)
  - gives field-to-field correlation
  - physically interpretable results with meaningful graphical displays
  - gives fundamental insight into physics
    - nonlinear dynamics
    - physical kinetics
    - quantization and second quantization
    - renormalization
    - topology
  - based on a transformation to a renormalization basis
    - dynamics is constrained to a low dimensional linear subspace of a complex Hilbert space (manifold) — the solutions to the Renormalization Group Equations, $f^{\beta(p)}$, with basis |β(p)>, a Reduced Order Model
    - a transformation from the cotangent bundle T^*M^N with a symplectic metric dπ^df to C^N with analyticity, Wigner-Weyl-like manifold-safe transformation
    - dynamics is geodesic motion determined by the topology of the low dimensional linear subspace C^N spanned by |β(p)>, determined by the analytic function H(z) with curvature given by dH/dβ (the S-Matrix or Heisenberg's Scattering Matrix)
- surrogate extrapolates well, if new physics (not captured by resistive MHD) is not becoming significant
- surrogate can be combined with experimental measurement to test causality hypothesis, and to characterize additional causality (model estimation), if needed
- emergent behavior of 2D MagLIF to self organized dipole state definitively demonstrated