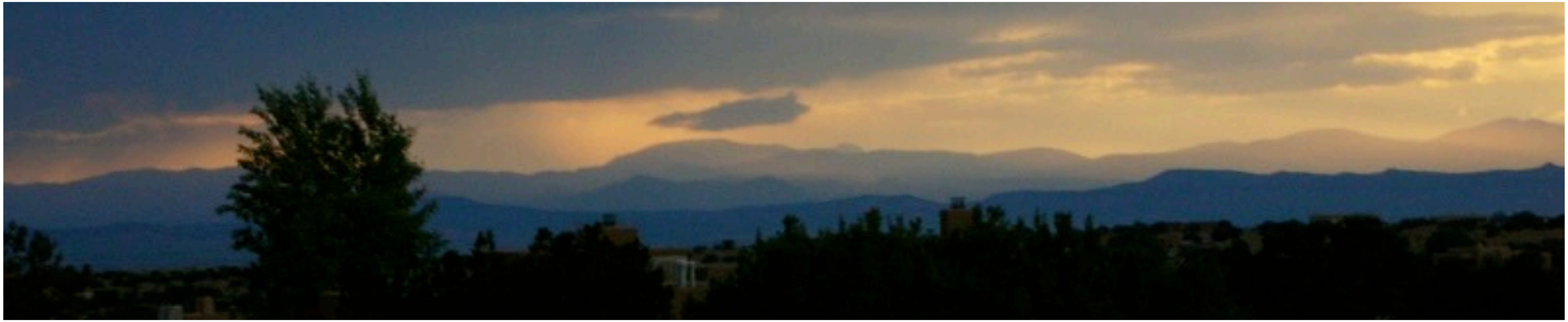

# A physics-based Reduced Order Model capturing the topology of dynamical manifolds

**Michael E. Glinsky**, *BNZ Energy Inc.*

BNZ Energy

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# Questions to ask ourselves

- About Artificial Intelligence:
  - What are the variables in the Reduced Order Models? Whether that be auto-encoders, U-nets, DQNs, or GPTs.
  - Our brain comes hardwired for the deep convolutional network. Why do we spend $10s millions to learn it on a case-by-case basis? What is this deep convolutional or functional transformation doing?
  - What is Q learning, that is what is Q? What flow is a generator generating, that is what transformation is generating the data?
  - Is Artificial Intelligence related to the topology and geometry of the system's dynamical manifold?
  - Is there a physical justification for ReLU?
  - Why is the logarithm so important?
  - Is there an equivalent of ponderomotive stabilization in Deep Reinforcement Learning?
- About Quantum Field Theory:
  - Why can only a few simple cases be renormalized and solved via the path integral approach to QFT? What is renormalization doing?
  - Why can gravitation and nonlinear dynamics be described as geodesic motion (minimum distance or action path) on a finite dimensional space, but QFT is described in a un-reconcilable statistical nature on a Hilbert space?
  - What are the Born Rule, Heisenberg's Uncertainty Principle, and First and Second Quantization really telling us?
  - There is no reality, really?

# The academic foundation on which this work is based

- mathematical foundations in:
  - Spivak's "Calculus on Manifolds", as sophomore undergraduate
  - AJ Lohwater's geometric perspective on complex analysis, that is the equivalency of an analytic function and the topology of its Riemann manifold, as sophomore undergraduate
  - studies on topological helicity invariants, as a graduate student under Michael Freedman
  - studies on Geometry of Physics, as a graduate student under Ted Frankel
- physics foundations in collective behavior:
  - studies on nonlinear dynamics and plasma physics from a canonical transformation perspective and Mayer Cluster Expansion perspective, as a UCSD graduate student under Tom O'Neil
  - studies on fundamental approach to plasma physics, as a graduate student under Marshall Rosenbluth
  - studies on statistical mechanics, as a graduate student under Duncan Haldane
  - studies on quantum field theory, as a adjunct faculty member at UWA under Ian McArthur

# Let us turn the shell over and examine field theory from a new perspective — the bottom up

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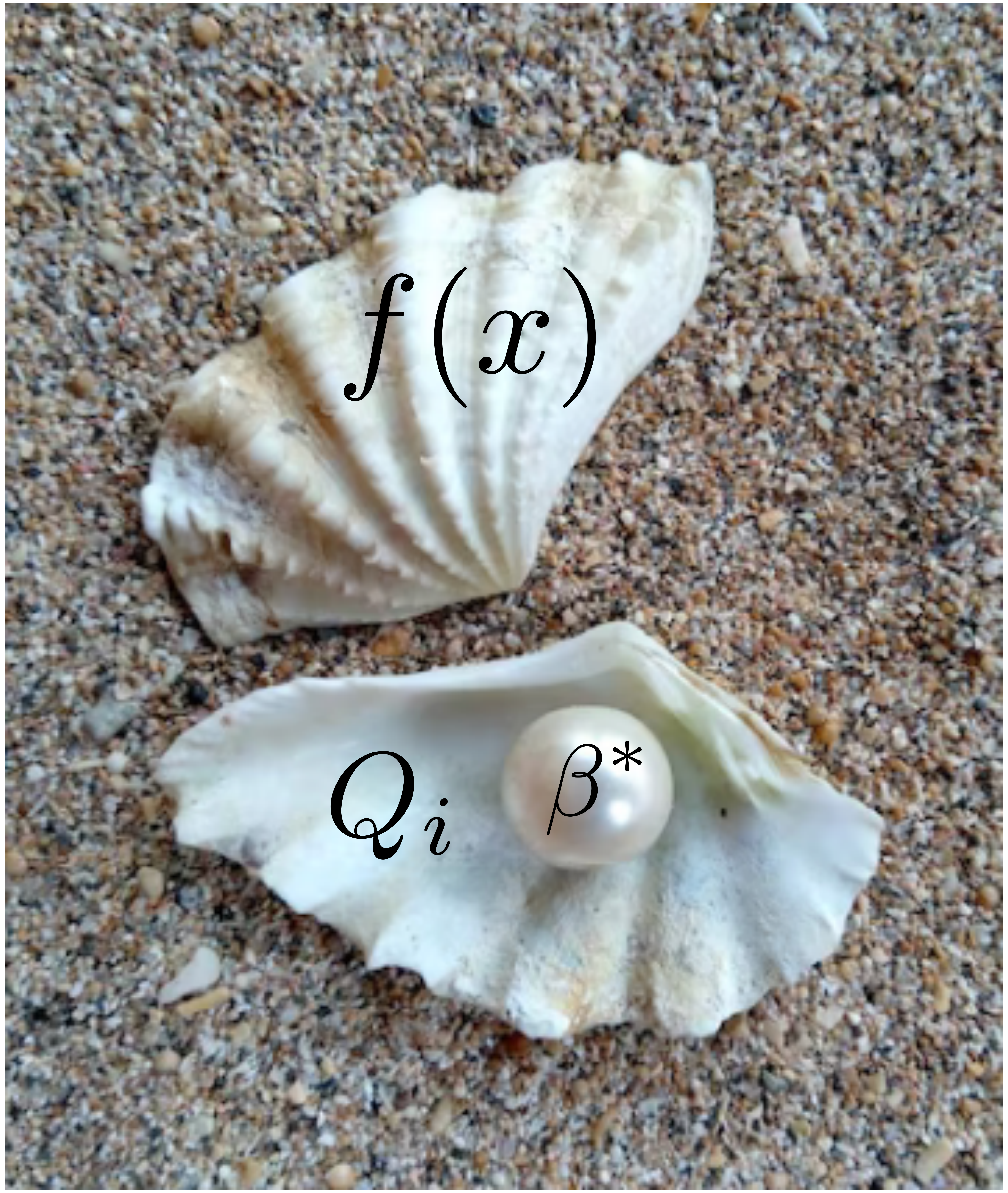
- From the top down, the traditional Quantum Field Theory perspective,
    - deeply convoluted fields, from an infinite dimensional Hilbert space of fields — the motion of the collective
    - fundamental singularities spewed everywhere
        - need to be consolidated by identifying fixed point and collecting the singularities by solving the Renormalization Group Equations for the “spectrums”
    - probability has a non-trivial distribution found by solving the Schrödinger Equation
- From the bottom up, the approach of plasma (collective) physics, deeply de-convolute the problem to the primal “spectral” domain of the individual where:
    - dimension is a small number
    - partial differential equations have been reduced to an algebraic expression (the analytic Hamiltonian), capturing the Lie group symmetries of the individual
    - singularities (of the analytic Hamiltonian) have been reduced to a few algebraic structures, capturing the geometry and topology of the dynamical manifold of the individual
    - probability is trivially distributed (Qs are uniformly distributed over 2π)



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# The geodesic puppet master of the collective

**collective**

$[\pi_i(x), f_i(x)]$

puppet master viewed via a "hall of mirrors"

**HST**

$S_p[f(x)]$

Heisenberg Scattering Transformation
canonical flow generating function**al**

Neural (HST) Operator (propagator or HJB)
or
Generative (HJB) Pretrained Transformer (HST)
or
Generative (HJB) Adversarial Network (HST)
or
Deep (HST) Q (HJB) Learning

**individual**

$(p_i, q_i)$

$$|\beta_i(z)\rangle = \frac{d^m H(\beta)}{d\beta_i^m} = S_i^m$$

(that is, **HST=HJB**)

**HJB**

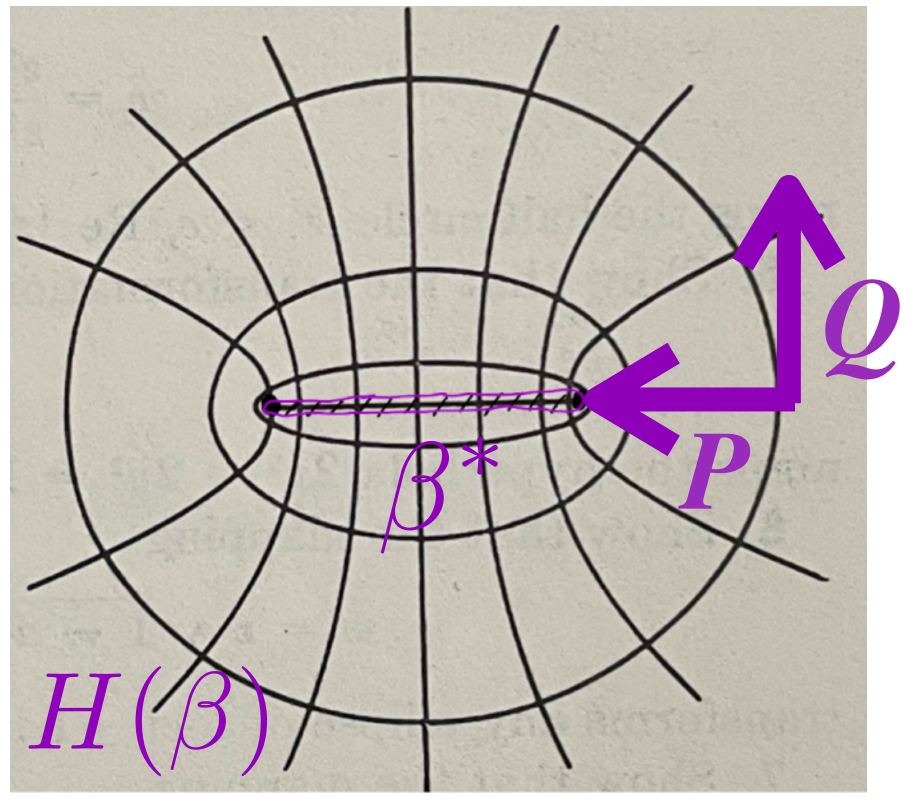
$$S_P(q) = \ln \rho_\theta(x) = \widetilde{V}_\theta(s)$$

generators DRL

canonical flow generating function
= action = entropy

**Q** puppet master

**P** external force




$m_Q \sim \omega_Q^{-2}$

$\omega_Q^* \to 0$

$m_Q^* \to \infty$

Q is immovable at β*

**geodesics**

$(P_i, Q_i)$

$$\frac{\partial S(P, E; q, \tau)}{\partial \tau} + H(\partial S/\partial q, q) - \nu\, S = 0$$

Hamilton-Jacobi-Bellman equation

Reduced Order Model (P,E;Q,τ)

# Mayer Cluster Expansion of collective systems or HST or Heisenberg's S-matrix or m-Body Scattering Cross Sections



$$S_p[f(x)]$$

Heisenberg Scattering Transformation (HST)
a logarithmic or canonical generating functio**nal**


$$p = \sum_{m=1}^{\infty} p_m$$

$p_{m+1} < p_m$ (Wick path ordering)

$$\ln(z) = \ln|z| + \mathrm{i}\,\arg(z)$$

$$\ln(R_0(z)) \xrightarrow[|z|\to 0]{} z$$

Glinsky and Maupin, Computational Mechanics **72**, 291, arXiv:2302.10243 (2023)

# Neural Network architecture to solve HJB for the generator of the canonical flow, a generating function

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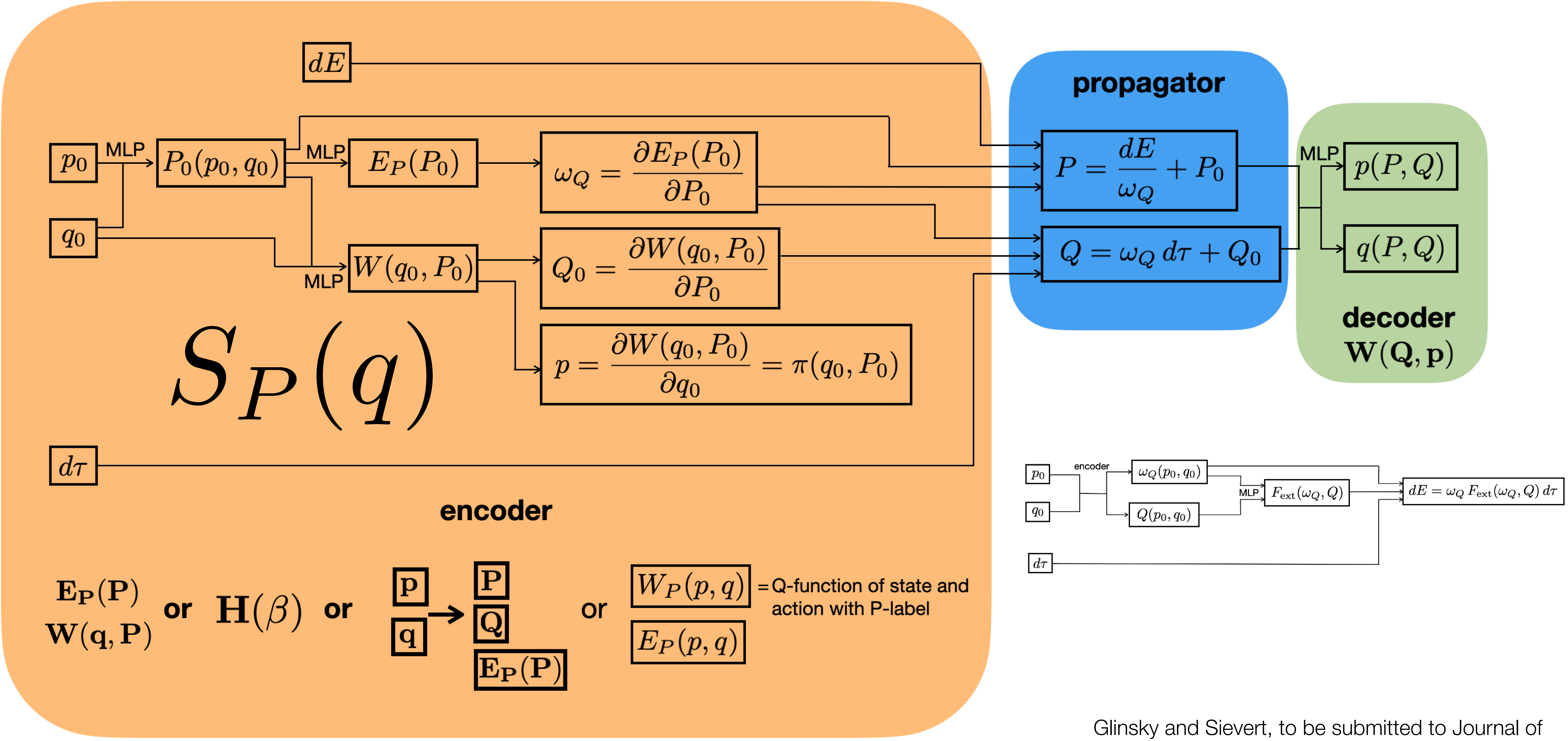


Glinsky and Sievert, to be submitted to Journal of Economic Affairs, arXiv:2310.04986 (2023)

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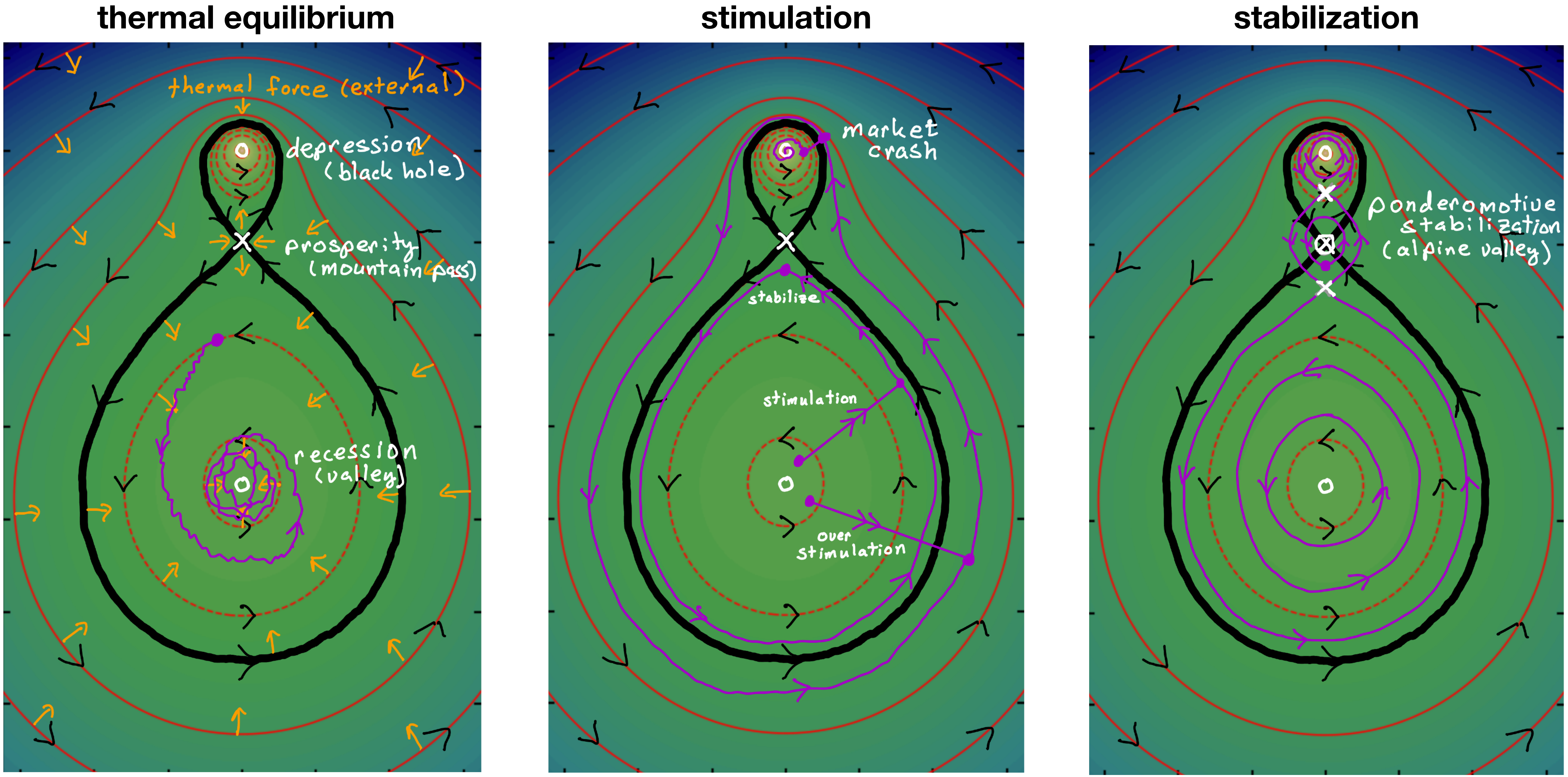
# Phase space evolutions



# Topological discovery of dynamical systems: "the collective behaves as one"



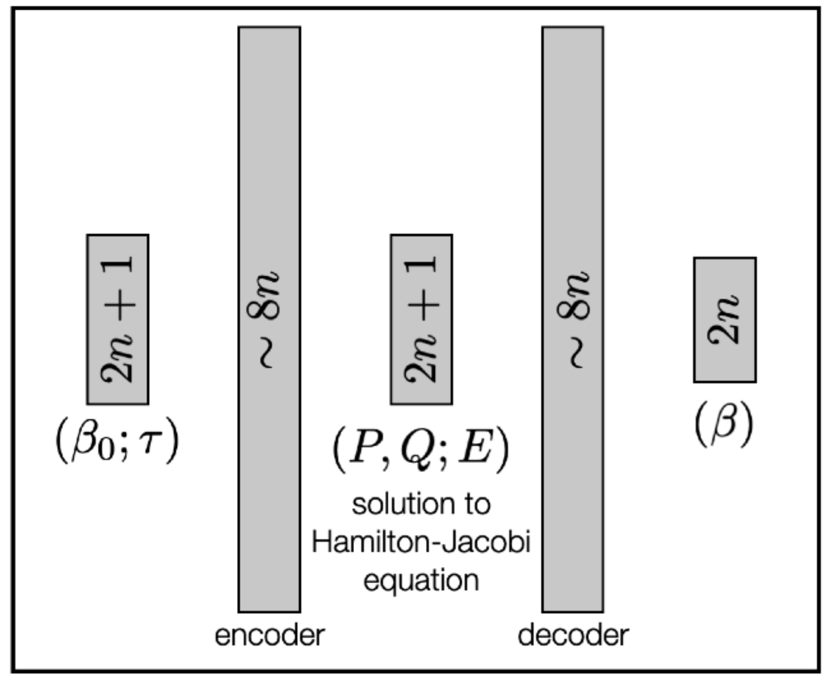
**ensemble of systems** or **field/fluid domain**

$[\pi_i(x), f_i(x)]$ → **deep deconvolution**: HST+PCA using local harmonic wavelet basis or local coherent states, with fast forward and inverse → **single system** or **S-matrix expansion** or **Mayer cluster expansion**: $|\beta_i(z)\rangle = S_{mi}$ → **decoder**: MLP with ReLU with encoder/decoder structure → **Lie group symmetries** or **geometry & topology**: $H(\beta)$ or $\beta^*$

motion on $T^*M^n$ (cotangent bundle) with symplectic metric $d\pi \wedge df$ and $H[\pi(x), f(x)]$ governed by Hamilton's equations

geodesic motion on $\mathbb{C}^n$ governed by Cauchy Riemann equations, equivalently the dimension and basis vectors (solutions to the Renormalization Group Equations) of the dynamical manifold

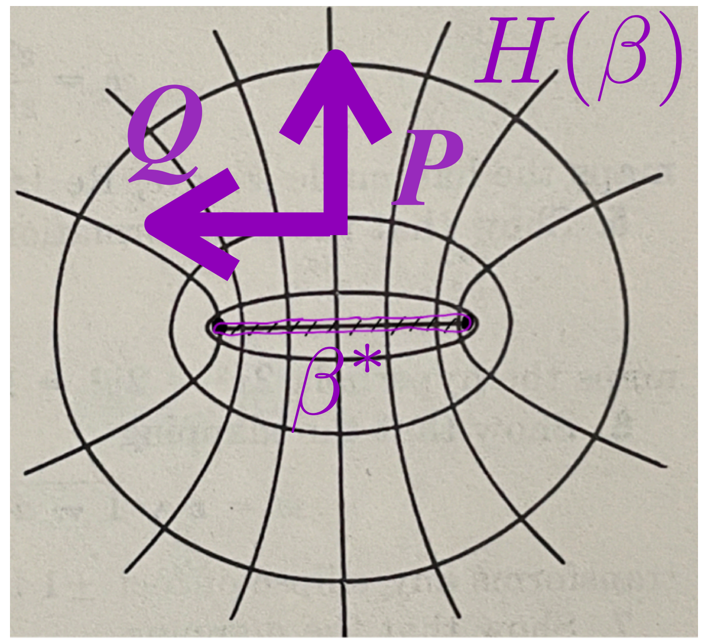
$$\mathrm{Re}(H(\beta_i = p_i + \mathrm{i}\, q_i)) = H(p_i, q_i)$$




analytic function that specifies geodesics and adjoints, equivalently the topology of the dynamical manifold

note: the **S-matrix** or complex curvature is

$$S_m = \frac{d^m H(\beta)}{d\beta^m}$$




**collective** $[\pi_i(x), f_i(x)]$ → **HST** $S_p[f(x)]$ → **individual** $(p_i, q_i)$ → **HJB** → $(P_i, Q_i)$

$$S_P(q) = \ln \rho_\theta(x) = \widetilde{V}_\theta(s)$$

generators DRL

canonical flow generator = action = entropy

Neural (HST) Operator (propagator or HJB)
or
Generative (HJB) Pretrained Transformer (HST)

note: **HST=HJB**

# Build-a-QFT: a simple beauty

**the evolution of THE individual**

**the evolution of the collective**

**pick-a-Hamiltonian**

$H(p_i, q_i) = \mathrm{Re}(H(\beta))$

(that is, pick a group symmetry, or conserved quantity)

→

**find-the-β***

$\mathrm{Im}(H(\beta))$ or $H(\beta)$

**HJB**

(that is, solve the HJB equation with MLP+ReLU, or the motion of the individual)

→

**find-the-perspective**

$$|\beta_i(z)\rangle = \frac{d^m H(\beta)}{d\beta_i^m} = S_i^m$$

(that is, that solve for the S-matrix, or Mayer cluster expansion)

→

**find-the-field**

**iPCA + iHST**

(that is, project and deeply convolve to field/fluid domain, or the motion of the collective)

→

→ **QFT**

profoundly beautiful is the equivalence of the Taylor expansion coefficients, that is S-matrix, and the complex d(ln(z)) symplectic structure, or analytic $T^2$ topology, to both the **HST** and **HJB**

# Heisenberg Scattering Transformation (HST) — logarithmic generating functional of the complex Hamiltonian

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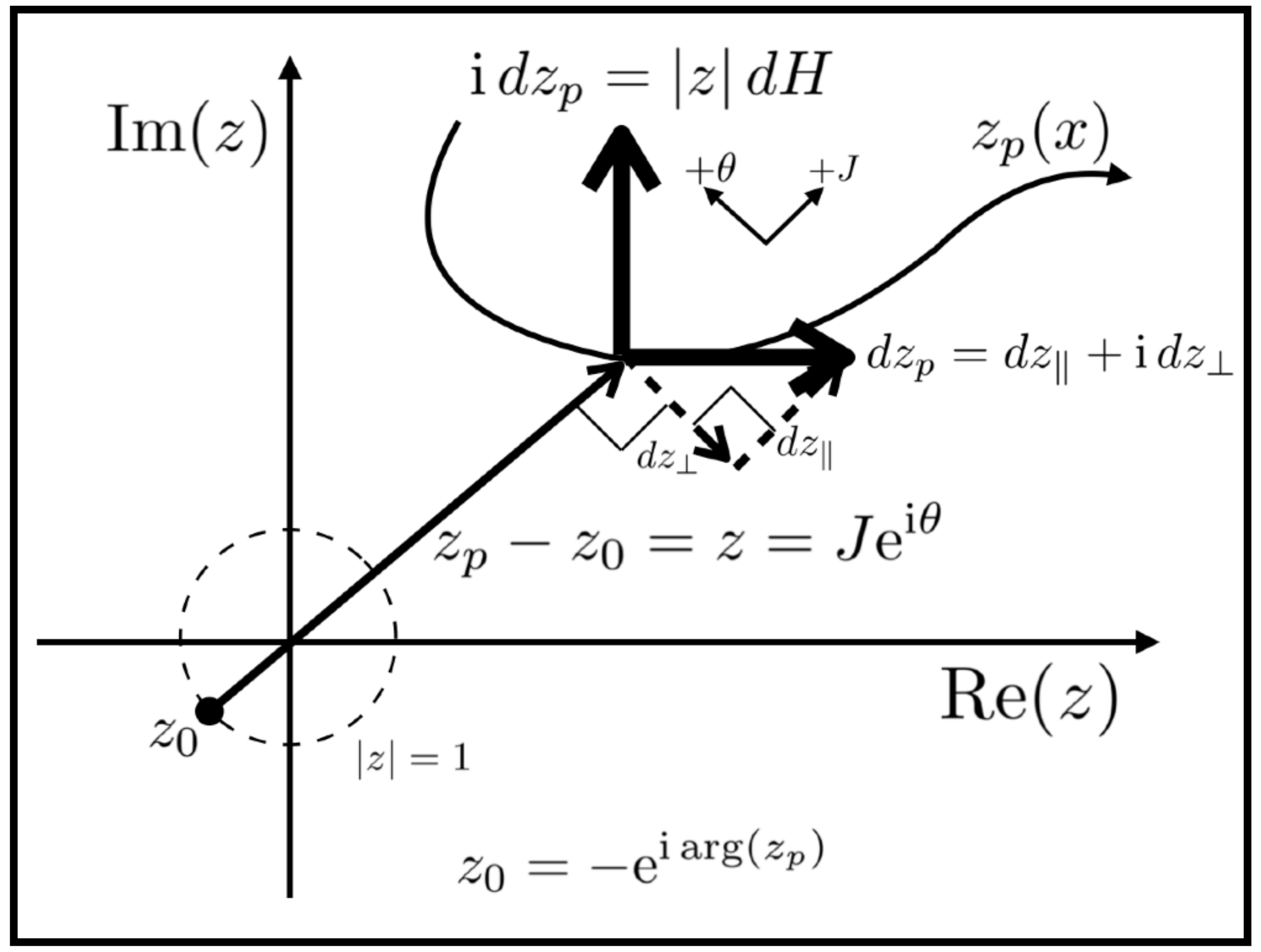


Taylor expand H(z)

$$\bar{H}(z) = \sum_{n=0}^{\infty} \frac{1}{n!} S_m(z_0)\,(z - z_0)^n$$

where

$$\bar{H} \equiv \mathrm{i}H$$

$$H_m(z(x)) = F[\pi(x), f(x)]$$

$$S_m(z) = \frac{d^m \bar{H}(z)}{dz^m} = \frac{dH_m}{dz}$$ **(= S-matrix)**

define the convolutional wavelet transform

$$\psi_p \star f(x) = \int \psi_p(x')\, f(x - x')\, dx'$$

and consider the analytic trajectory

$$z_p(x) \equiv \psi_p \star H_{m-1}(z(x))$$

take the covector along this trajectory

$$dz_p$$

change to radial coordinates

$$z_0 = -\mathrm{e}^{\mathrm{i}\arg(z_p)}$$

$$z = z_p - z_0 = J\mathrm{e}^{\mathrm{i}\theta}$$

rotate by π/2 to get the covector

$$dH_m(z) = \mathrm{i}\frac{dz_p}{|z|} = \mathrm{i}\left(\frac{dz_\parallel}{|z|} + \mathrm{i}\frac{dz_\perp}{|z|}\right)$$

$$= \mathrm{i}\left(\frac{dJ}{J} + \mathrm{i}\theta\right) = \mathrm{i}\, d(\ln(J\mathrm{e}^{\mathrm{i}\theta}))$$

$$= \mathrm{i}\, d(\ln z) = d(\mathrm{i}\,\ln z).$$

with the definition

$$R_0(z) \equiv z + \mathrm{e}^{\mathrm{i}\arg(z)}$$

get the **recursion relation**

$$H_m = \mathrm{i}\,\ln z = \mathrm{i}\,\ln(R_0(z_p))$$

$$= \mathrm{i}\,\ln R_0\, \psi_{p_m} \star H_{m-1}.$$

so that

**Heisenberg Scattering Transformation**

$$H_m[f(x)](z) = \phi_{px} \star \left(\prod_{k=0}^{m} \mathrm{i}\ln R_0 \psi_{p_k} \star\right) \mathrm{i}\ln R_0 f(x),$$

$$\ln(z) = \ln|z| + \mathrm{i}\,\arg(z)$$

$$\ln(R_0(z)) \xrightarrow[|z|\to 0]{} z$$

$$\xrightarrow[|z|\to\infty]{} \ln|z| \quad (\log(\mathrm{MST})\ \mathrm{MHD\ example})$$

$$\xrightarrow[|z|\to\infty]{} \ln|z|\,\mathrm{e}^{\mathrm{i}\arg(z)/\ln|z|}$$

$$\sim \sum_{k=0}^{n} |z|\,\mathrm{e}^{\mathrm{i}\,k\,\arg(z)} \quad (\mathrm{Wavelet\ Phase\ Harmonics})$$

$$\xrightarrow[n=0]{} |z| \quad (\mathrm{Mallat\ Scattering\ Transformation,\ MST})$$

similar to ReLU

# MLP solves Hamilton-Jacobi (Bellman) or Laplace equation for time invariant coordinates or geodesics

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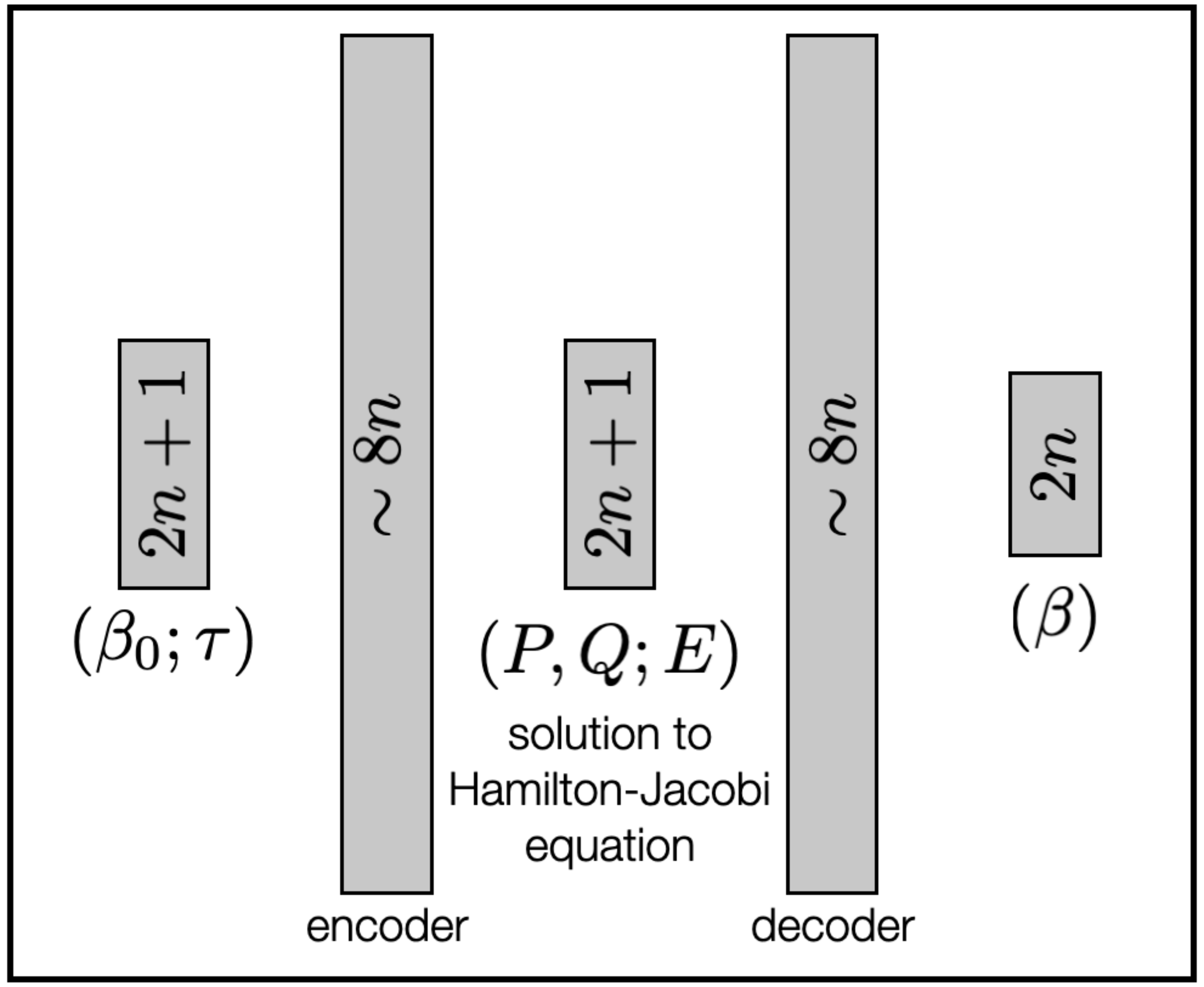
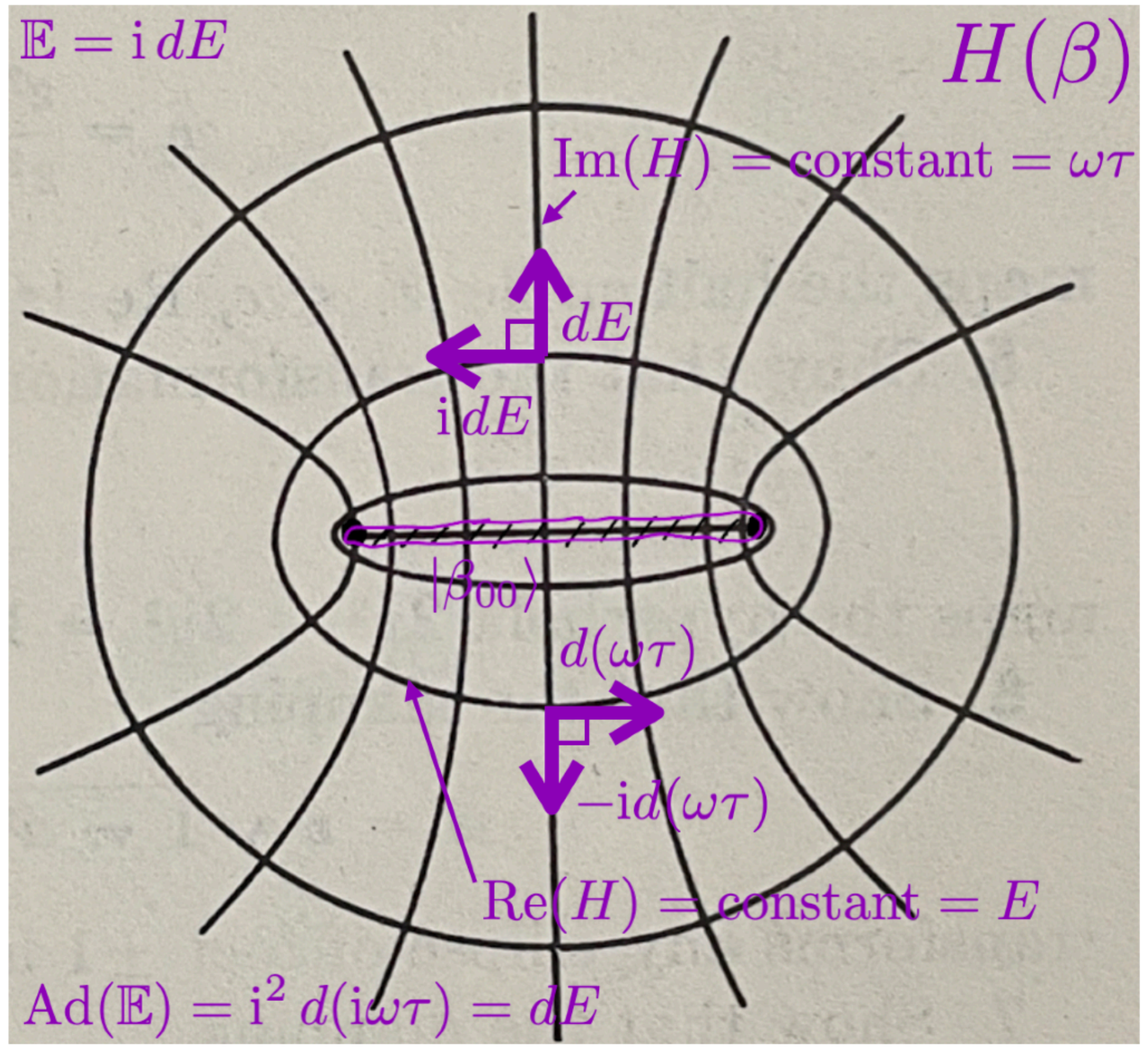



$$\mathbb{H} = \mathbb{E}\otimes \mathrm{Ad}(\mathbb{E}) = H \otimes \mathrm{Ad}(H) = H\otimes dH$$




$$H(\beta) = E + \mathrm{i}\omega\tau = E_P(P) + \mathrm{i}Q$$

$$\text{group parameter} = \tau + \mathrm{i}E/\omega$$

$$d(\omega\tau) = \mathrm{i}dE \text{ (Cauchy-Riemann or Hamilton's equations)}$$

$$\frac{\partial S(q,t)}{\partial t} + H\left(\frac{\partial S(q,t)}{\partial q}, q, t\right) = 0$$ (Hamilton-Jacobi Equation)

$$\frac{\partial V(q,t)}{\partial t} + \frac{\partial V(q,t)}{\partial q} f(\partial V(q,t)/\partial q, q, t) + C(\partial V(q,t)/\partial q, q, t) = 0$$

(Hamilton-Jacobi-Bellman Equation)

$$S(q,t) = \text{action functional} = \text{generating function of canonical coordinate transformation}$$

$$H(\beta) = (\beta + 1/\beta)/2 = (\beta + \mathrm{i})(\beta - \mathrm{i})/2\beta$$

$$n = n_g = 1$$

two sheeted Riemann surface or manifold with branch cut between -1 and 1

$$\omega = d\pi\wedge df - dH\wedge d\tau = d\lambda = d(\pi\, df - H\, d\tau) = 2\, d\tau \wedge dH$$

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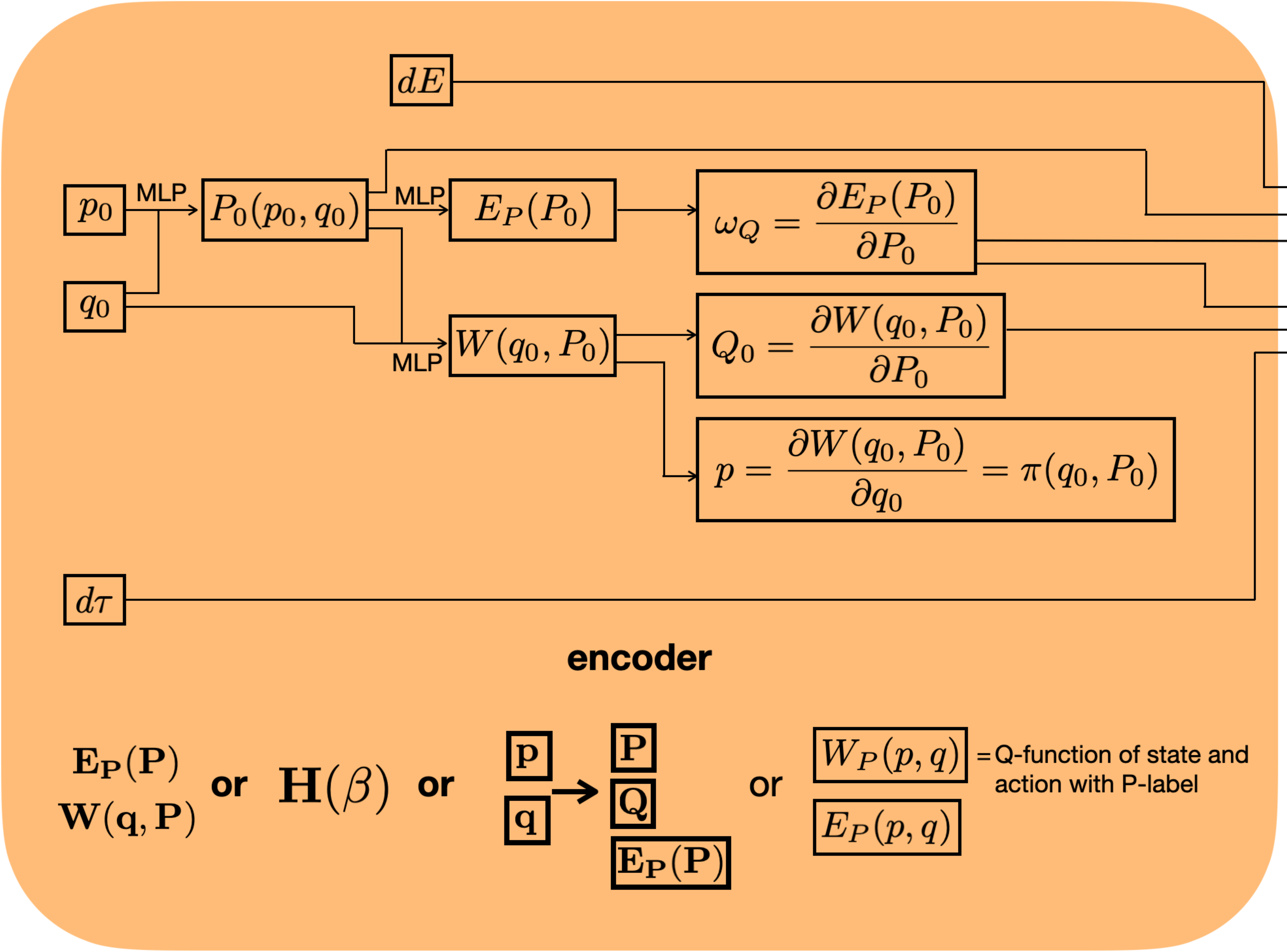
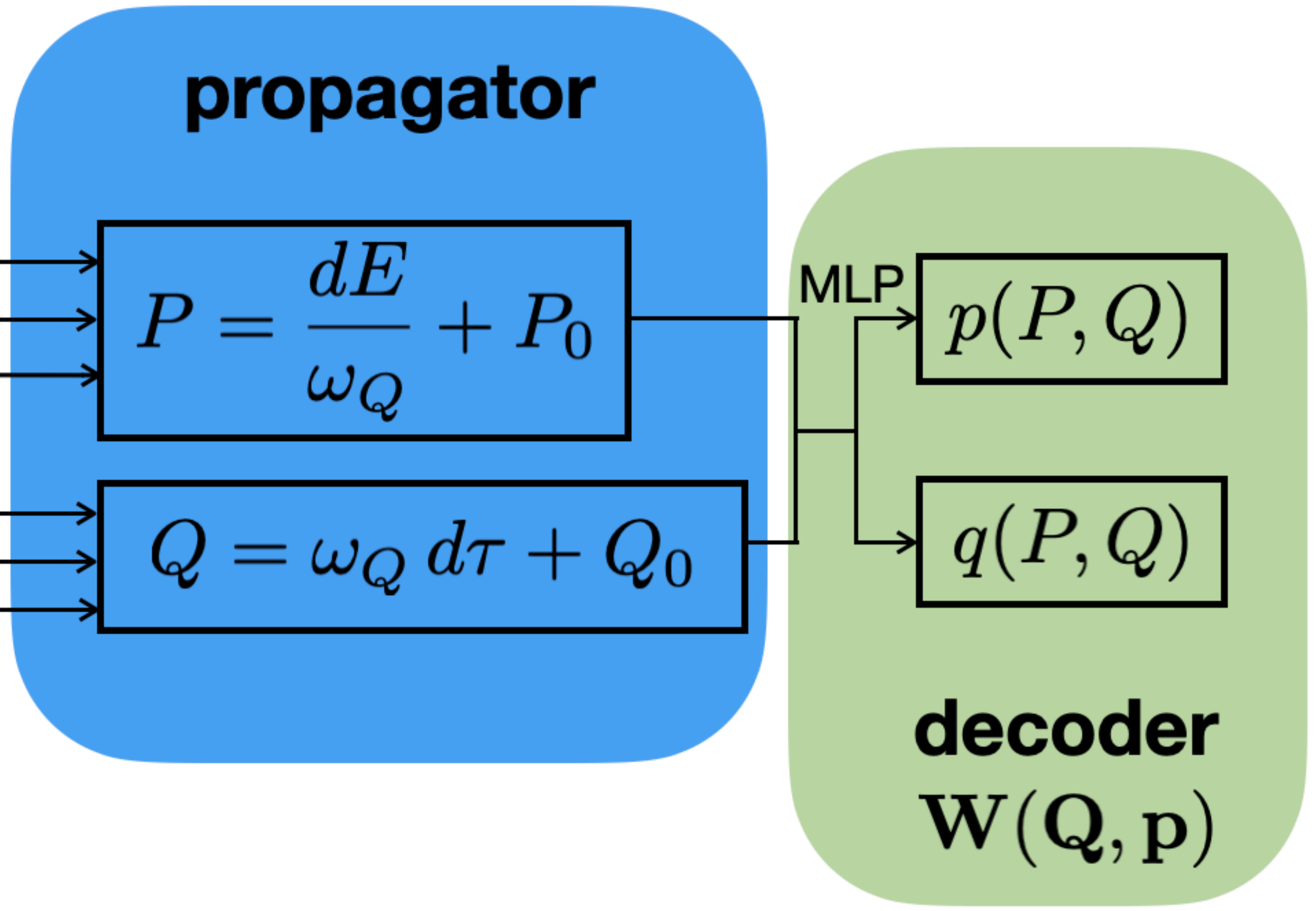
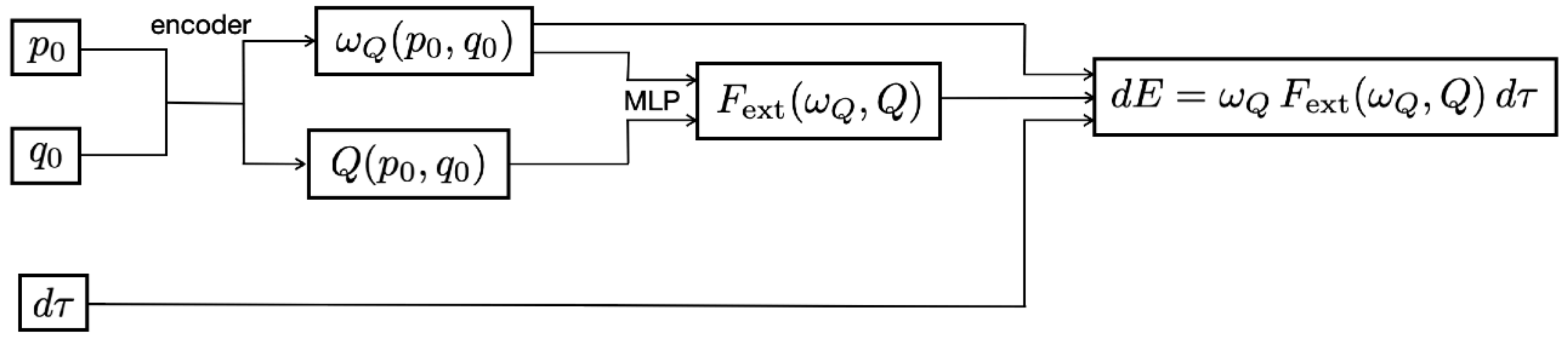
# Neural Network architecture to solve HJB




Glinsky and Sievert, to be submitted to Journal of Economic Affairs, arXiv:2310.04986 (2023)

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# General surrogate MLDL architecture (generator)

# General MLDL architecture to control complex systems (Deep Reinforcement Learning)

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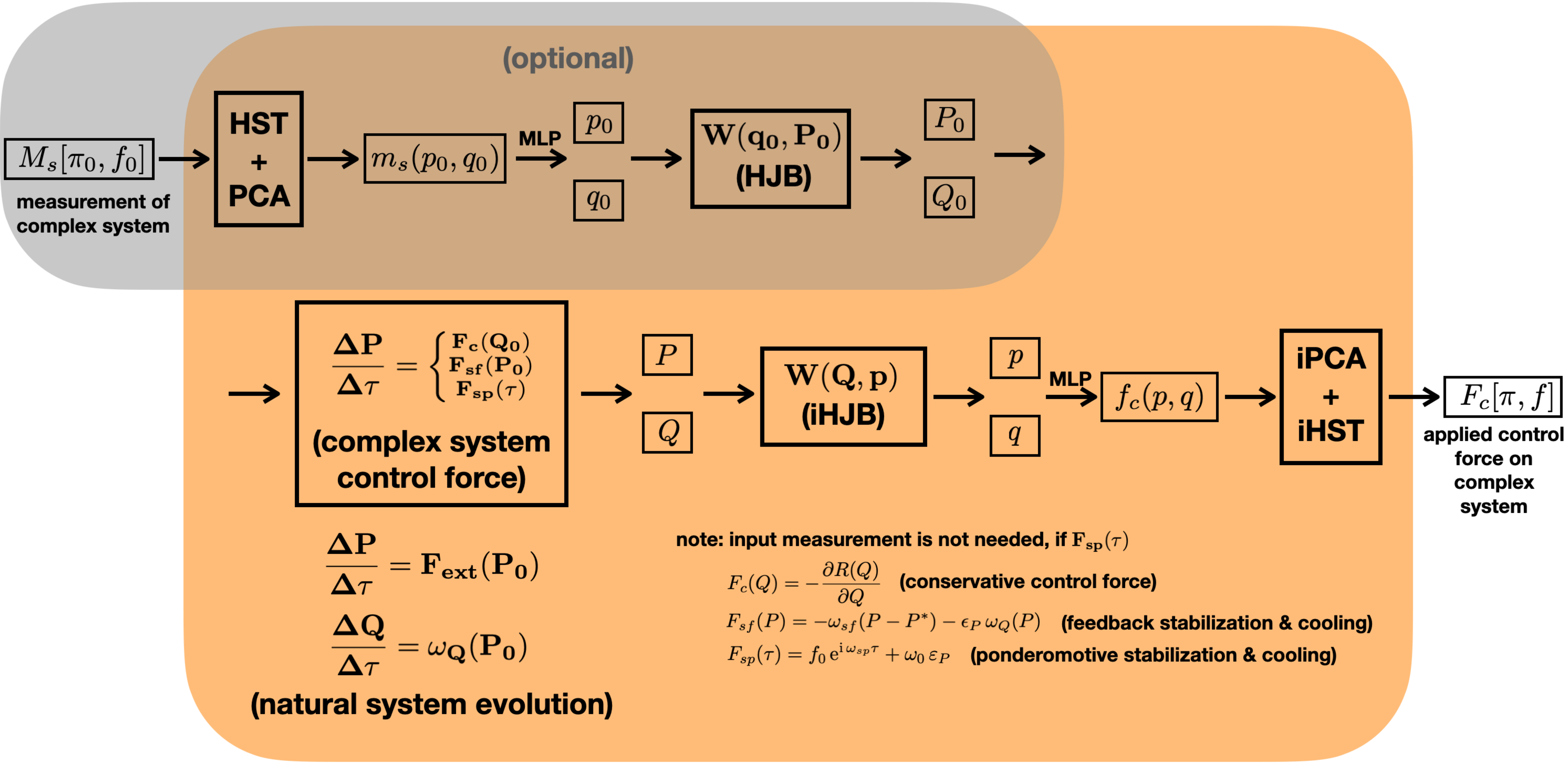

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# A simple analytic example of H(z)

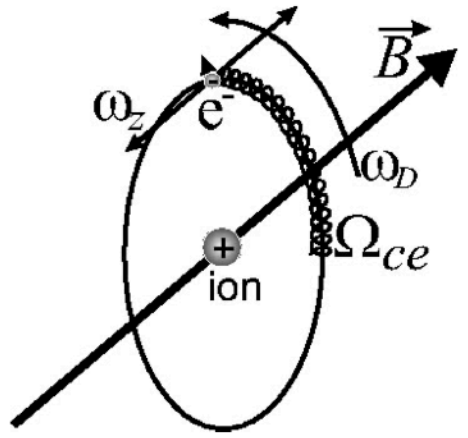
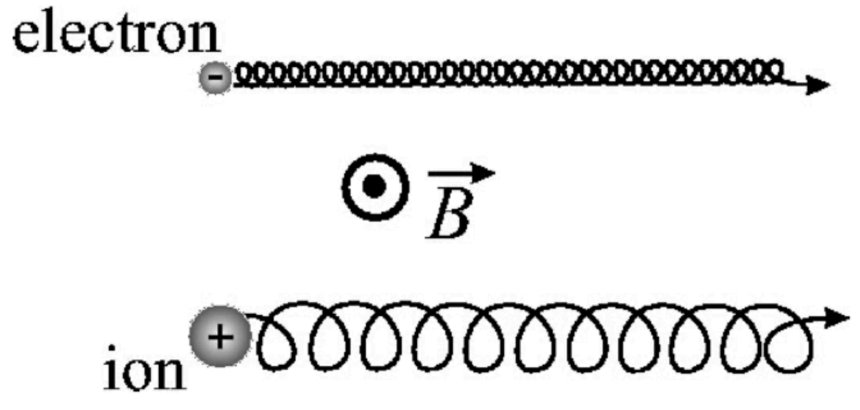



FIG. 1. Drawing of guiding center atom. In order of descending frequency, electron executes cyclotron motion, oscillates back and forth along a field line in the Coulomb well of the ion, $\vec{E}\times\vec{B}$ drifts around the ion.

**guiding center atom**




**drifting pair**

Start with full dynamics for ion and GC dynamics for electron in constant magnetic field

$$\hat{H}_2 = \frac{\vec{p}_i - \frac{e}{c}\vec{A}(\vec{x}_i)}{2m_i} + \frac{p_{ze}^2}{2m_e} - \frac{e^2}{\sqrt{(x_i-x_e)^2+(y_i-\frac{p_{xe}}{m_e\Omega})^2+(z_i-z_e)^2}}$$

use $\vec{A}(\vec{r}) = \frac{1}{2}\vec{B}\times\vec{r}$ with $\vec{B} = B\hat{z}$

Applying several canonical transformations the Hamiltonian can be written as

$$H_2(R;\psi,p_\psi,z,p_z) = \Omega_i\left[m_e\Omega\frac{R^2}{2} - R\sqrt{2m_e\Omega\, p_\psi}\cos\psi + p_\psi\right] + \frac{p_z^2}{2\mu} - \frac{e^2}{\sqrt{\frac{2p_\psi}{m_e\Omega}+z^2}}$$

E×B drift dynamics

Kuzmin, O'Neil and Glinsky, Phys. Plasmas **11**, 2382 (2004)
Glinsky and O'Neil, Phys. Fluids B **3**, 1279 (1991)

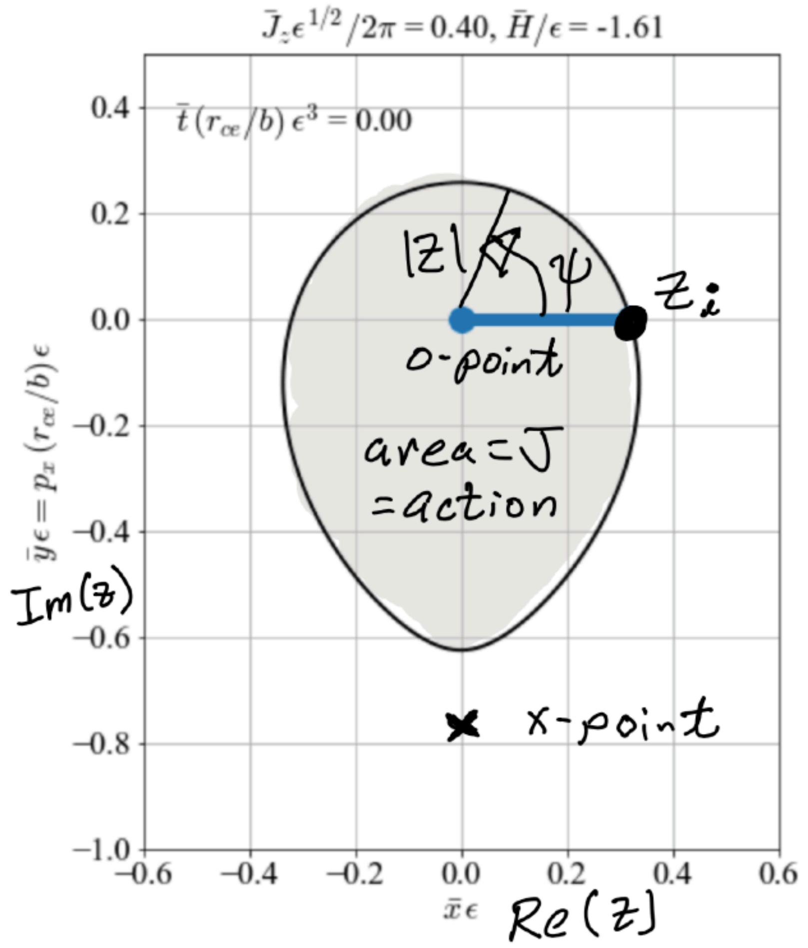
This can be reduced to a 1D problem by averaging over the z-motion

$$H_z(R,J_z;\bar{\psi},\bar{p}_\psi) = \Omega_i\left[m_e\Omega_e\frac{R^2}{2} - R\sqrt{2m_e\Omega_e\bar{p}_\psi}\cos\bar{\psi} + \bar{p}_\psi\right] + H_1(J_z,\bar{p}_\psi)$$

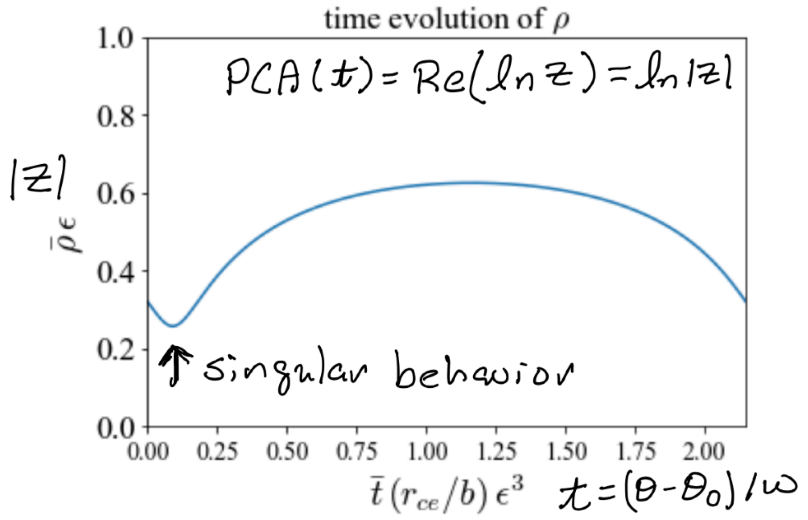
now defining $z^*z \equiv \frac{2\bar{p}_\psi}{m_e\Omega\bar{b}}$, $\frac{1}{a} \equiv \left(\frac{b}{r_{ce}}\right)^2\frac{m_e}{m_i}$, $\frac{R}{a} \equiv \left(\frac{b}{r_{ce}}\right)\left(\frac{v_{i\perp}}{v_e}\right)$, $\bar{J}_z \equiv \frac{J_z}{m\bar{v}_e\bar{b}}$

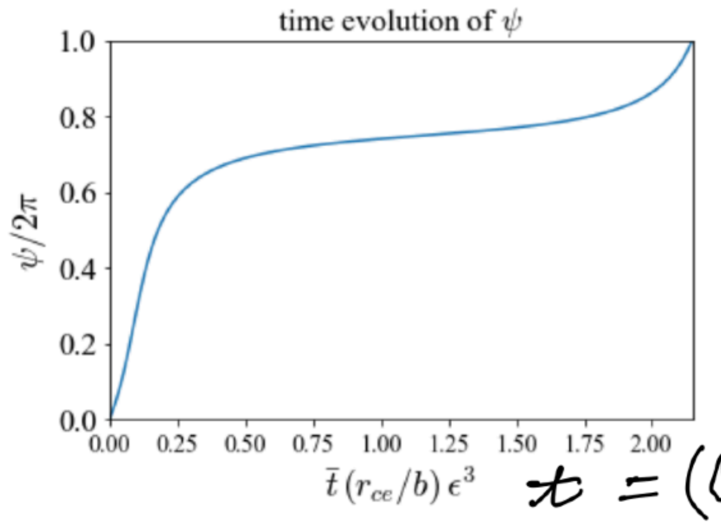
$z_0^* z_0 \equiv R^2$ $\quad \bar{H} = H_1/k_B T_e$

$$\bar{H}(C;z) = \bar{H}(\bar{J}_z,a,z_0;z) = \frac{1}{2}|z-z_0|^2 + a\bar{H}_1(\bar{J}_z,|z|)$$
$$C = (a, z_0, \bar{J}_z) \qquad = \frac{1}{2}(z-z_0)^*(z-z_0) + a\bar{H}_1(\bar{J}_z,\sqrt{z^*z})$$




$\psi = \arg(z)$
$\rho = |z|$



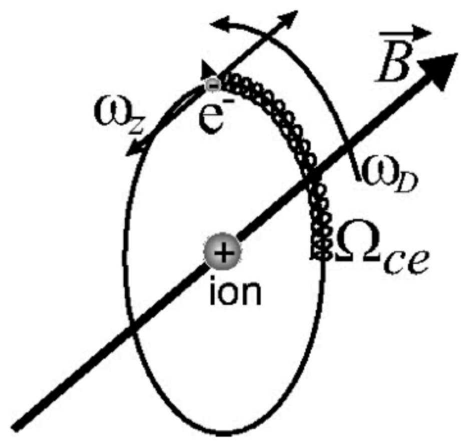




$\arg(z)/2\pi = \mathrm{Im}(\ln z)/2\pi$

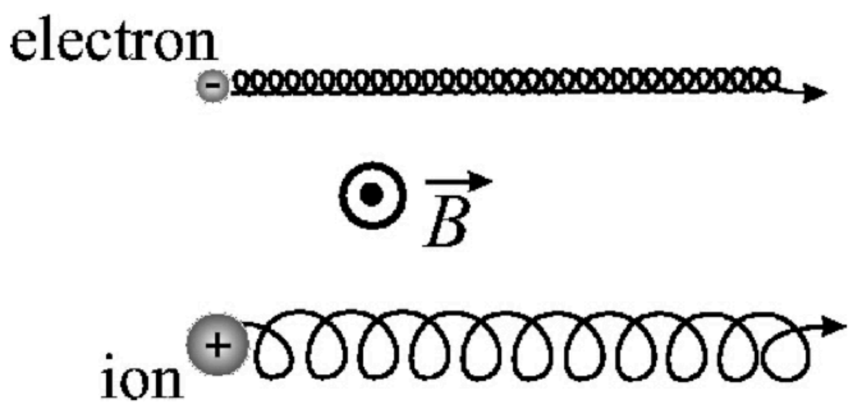
$t = (\theta - \theta_0)/\omega$

http://qitech.biz/videos/phase_space.html

# Phase space dispersion

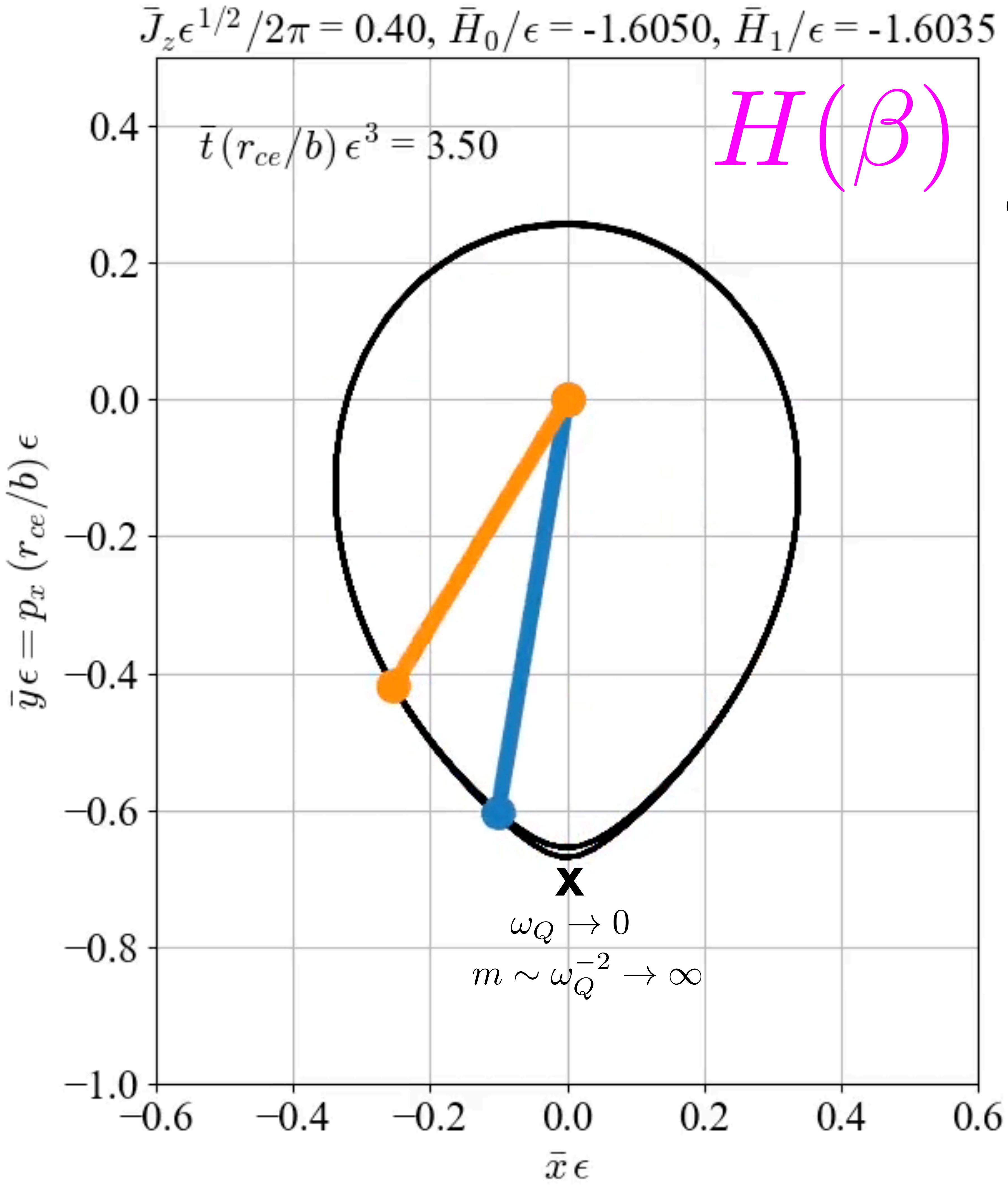



**guiding center atom**




**drifting pair**

**guiding center atom**




$\bar{H}_s/\epsilon = -1.6022$

qitech.biz/videos/phase_space_2.html

$\omega_{Q0} = 2.33$
$\omega_{Q1} = 2.03$

Glinsky and O'Neil, Phys. Fluids B **3**, 1279 (1991)
Kuzmin, O'Neil and Glinsky, Phys. Plasmas **11**, 2382 (2004)

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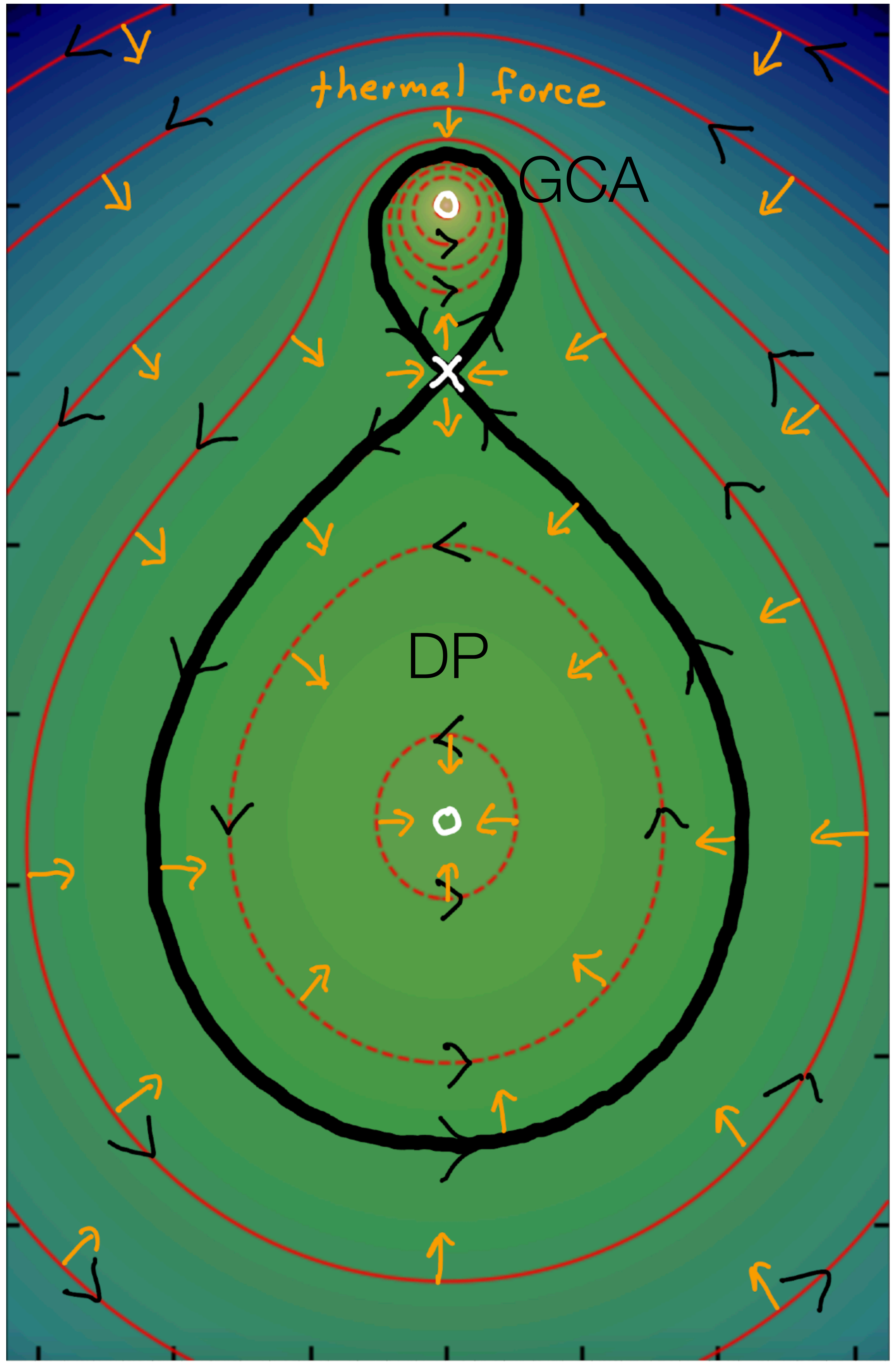
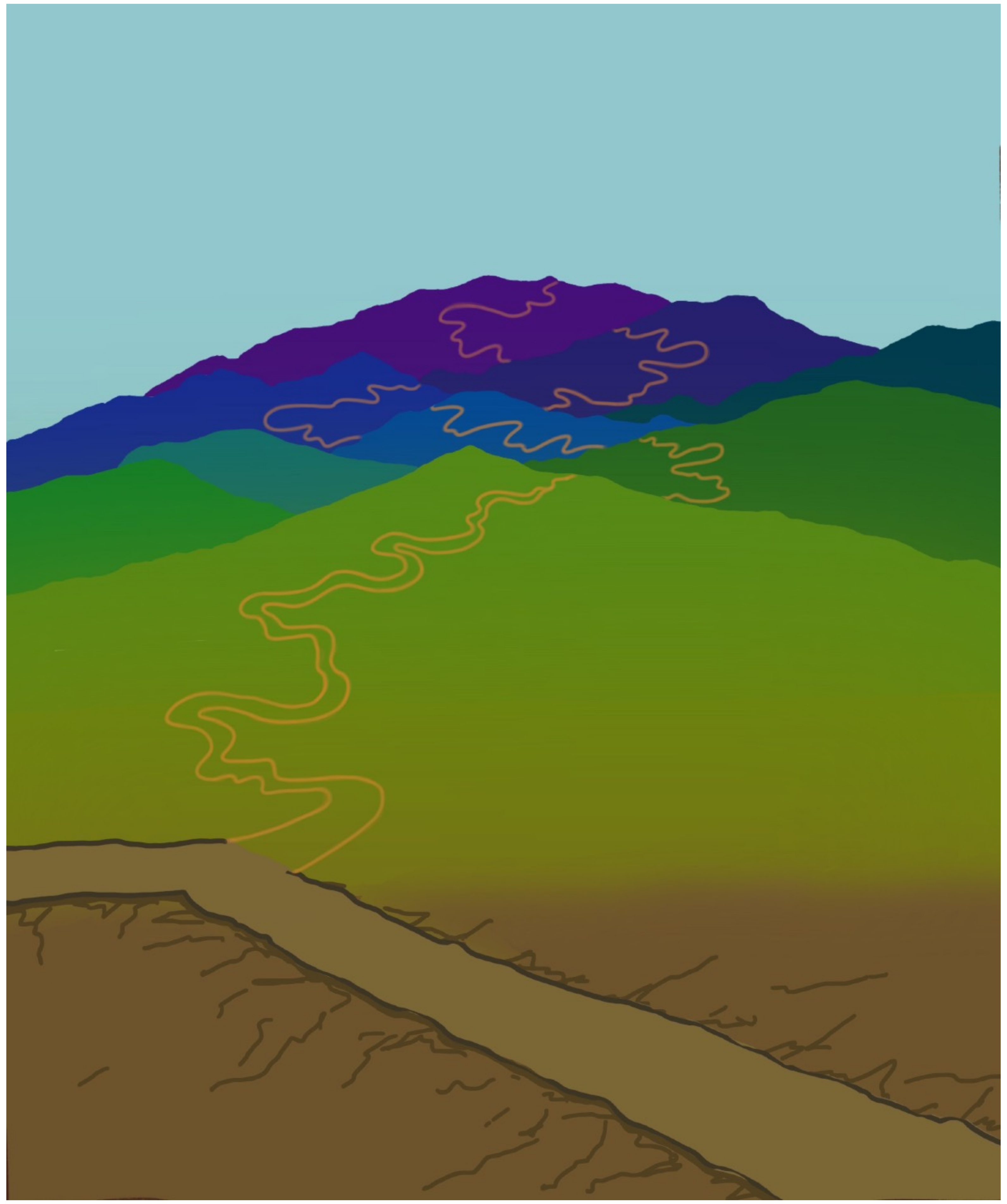
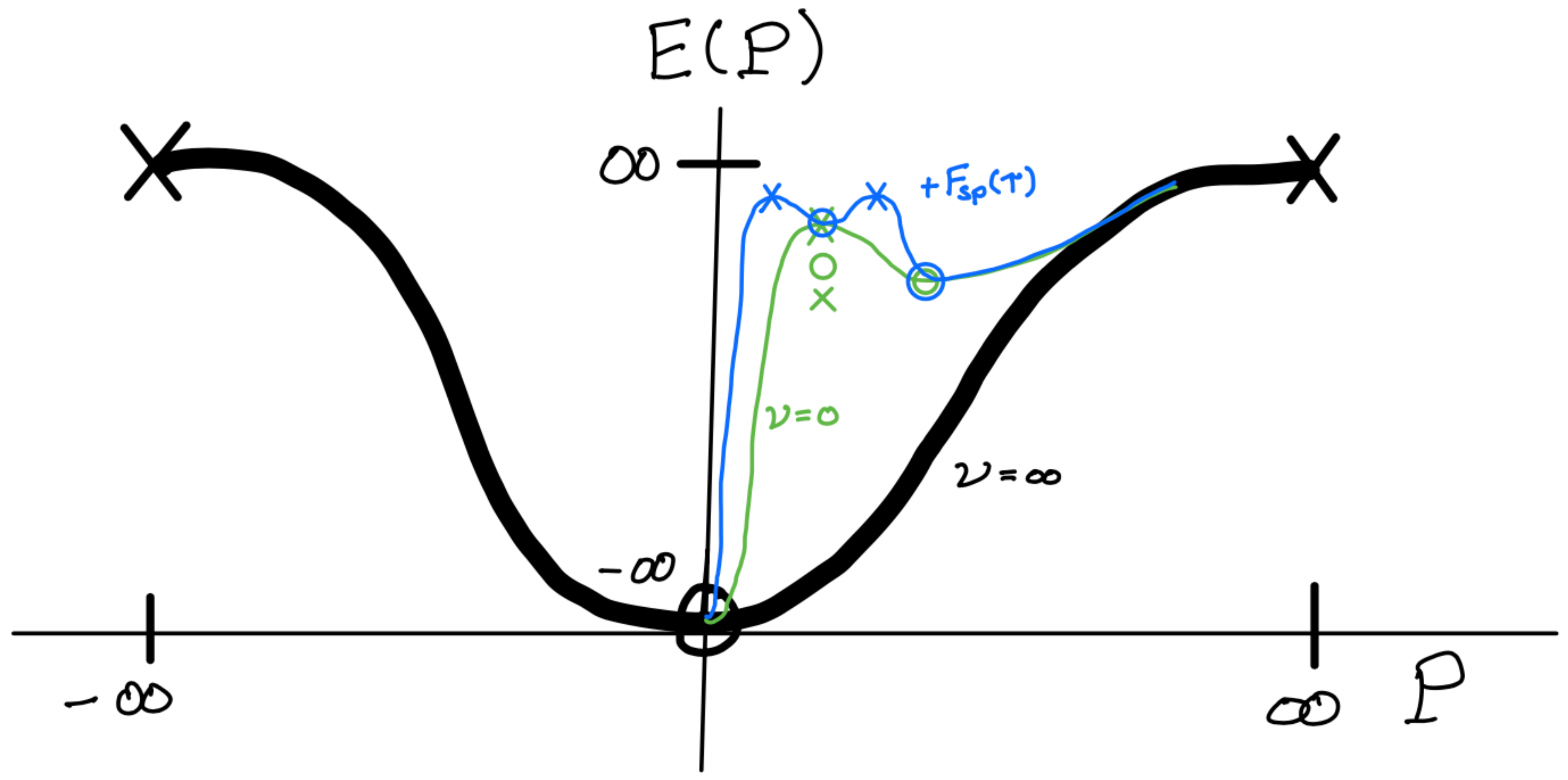
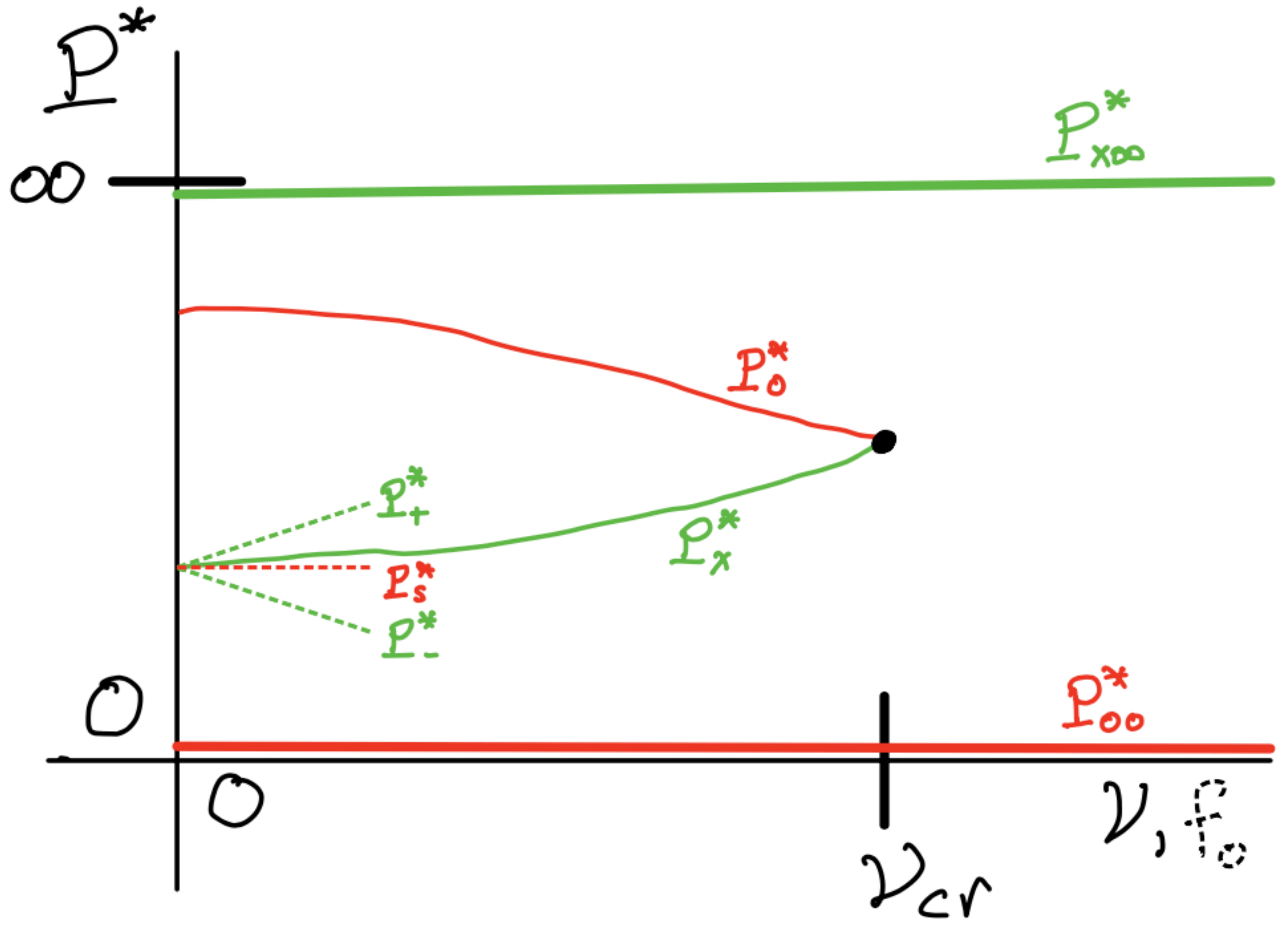
# Ponderomotive stabilization of saddle point

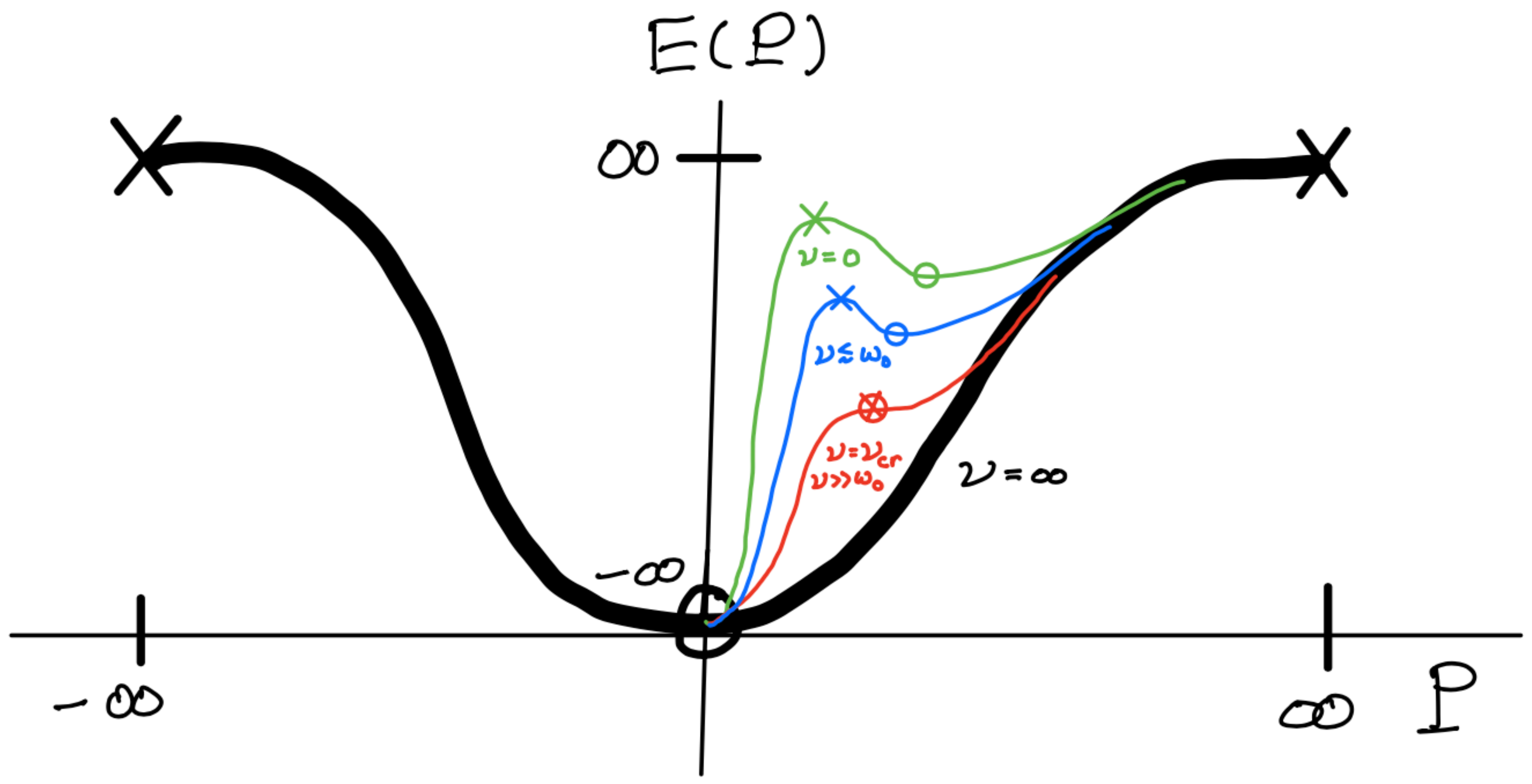


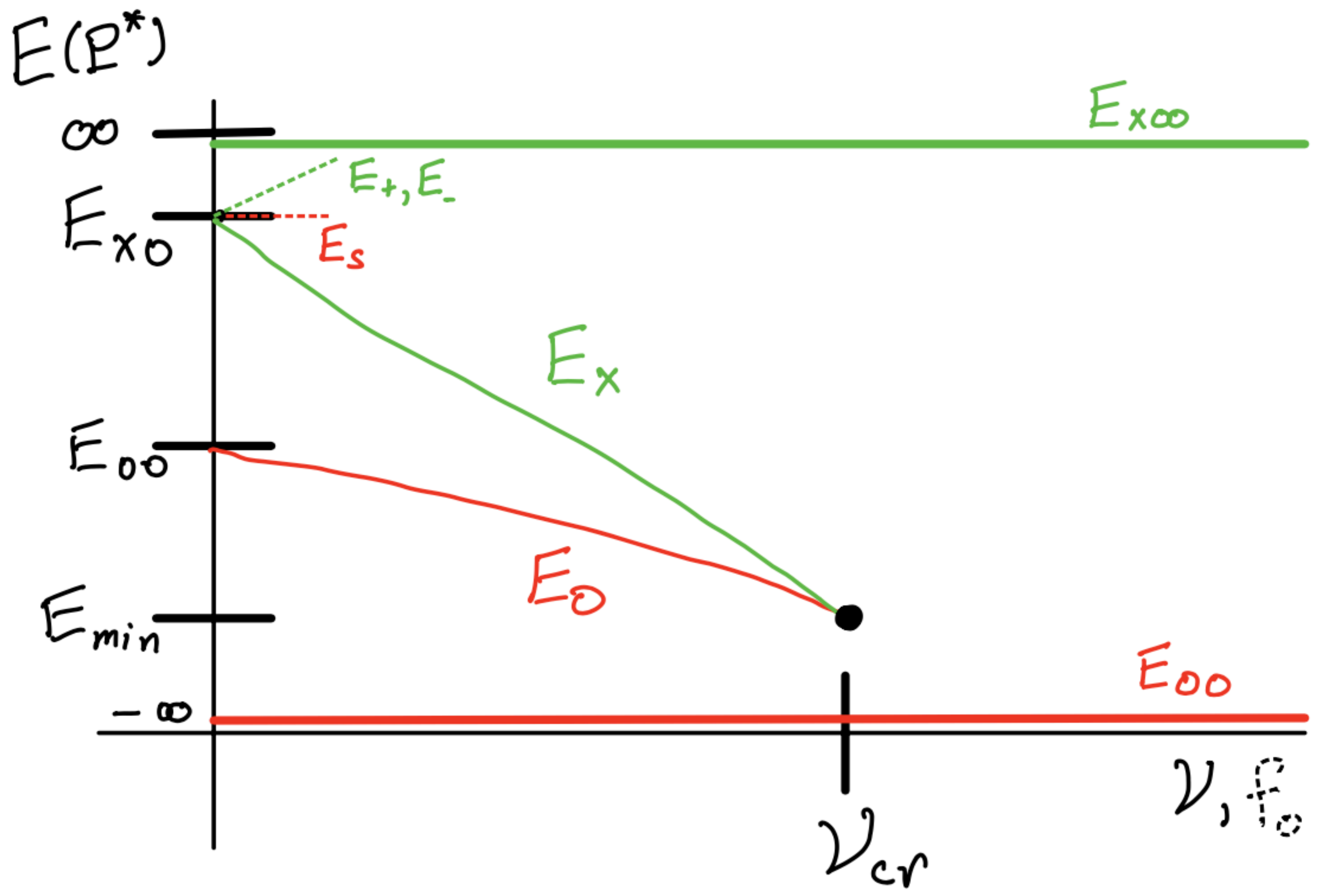



# Topological and geometric manipulation for control



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# What is Q learning?

$$\widetilde{Q}(s,a;\theta) = W(q, P(p,q)) \equiv W_P(p,q) = W(q,p;P)$$

**action** or **Hamilton's characteristic function** or **generating function of canonical transformation solving HJB**

$$\widetilde{V}(s;\theta) = W(q,P)$$

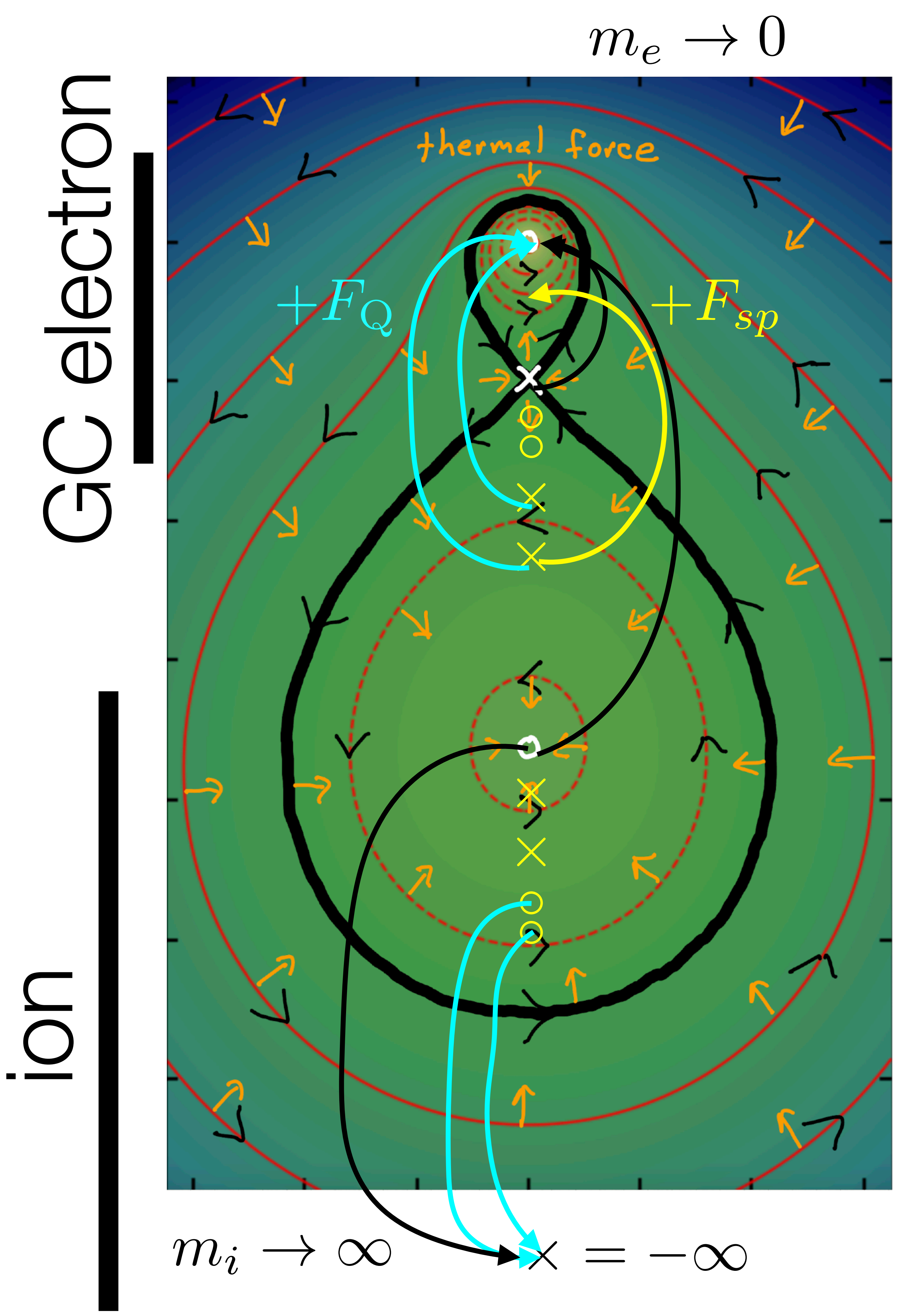
$$\widetilde{\pi}(s;\theta) = \frac{\partial W(q,P)}{\partial q} = \pi(q,P) = p$$

$$Q = \frac{\partial W(q,P)}{\partial P} = \frac{\partial \widetilde{Q}(s,\theta)}{\partial \theta}$$

$$H(\beta) = H(P,Q) = E_P(P) + \mathrm{i}\,Q$$

**analytic continuation of the real Hamiltonian** or **imaginary part of the complex Hamiltonian**

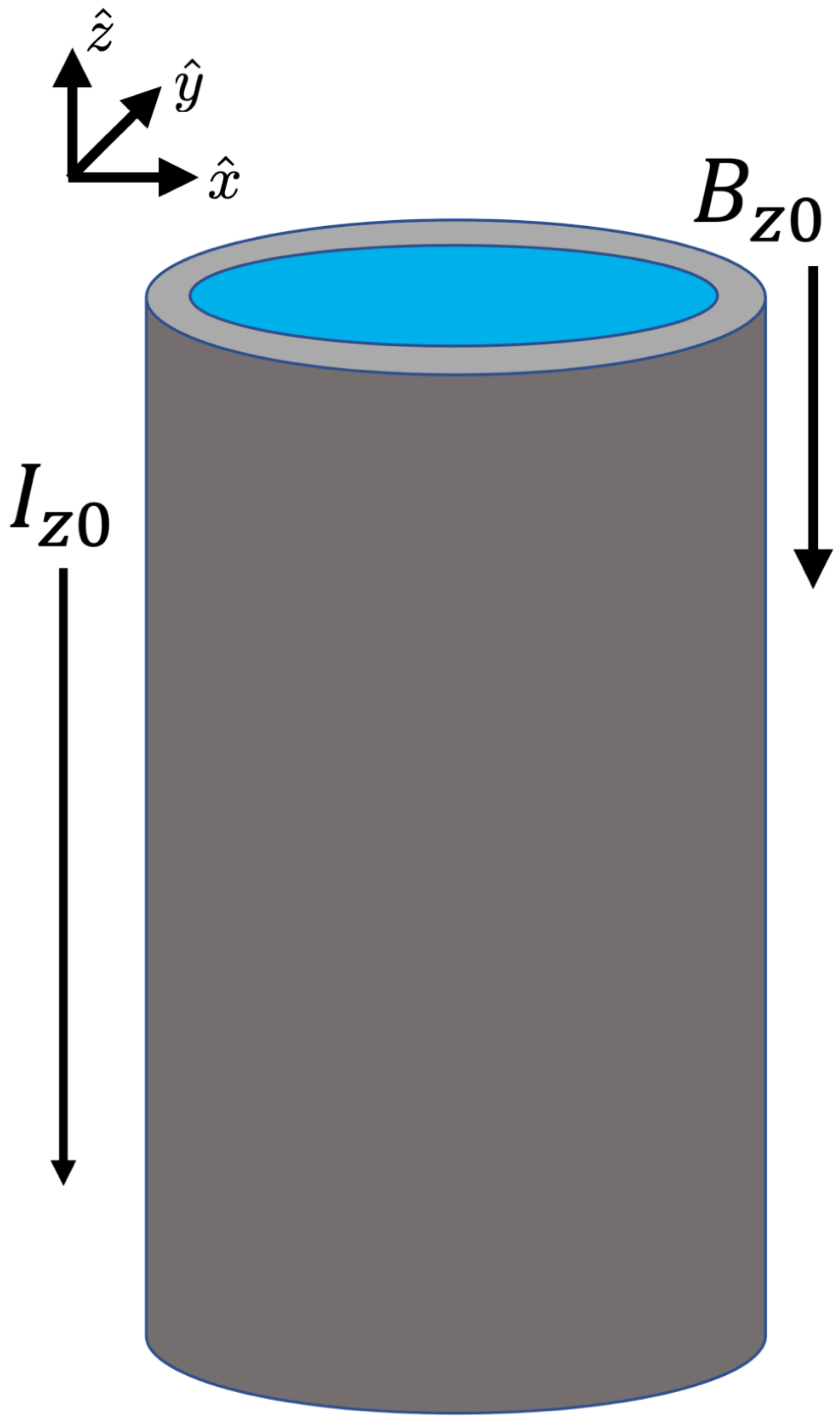
$$\omega_Q = \frac{\partial E_P(P^*)}{\partial P} = 0 \qquad \begin{cases} V^*(s) = \widetilde{V}(s,\theta^*) = W(q,P^*) = V^*(q) \\ \pi^*(s) = \widetilde{\pi}(s;\theta^*) = \dfrac{\partial W(q,P^*)}{\partial q} = \pi(q,P^*) = \pi^*(q) \end{cases}$$

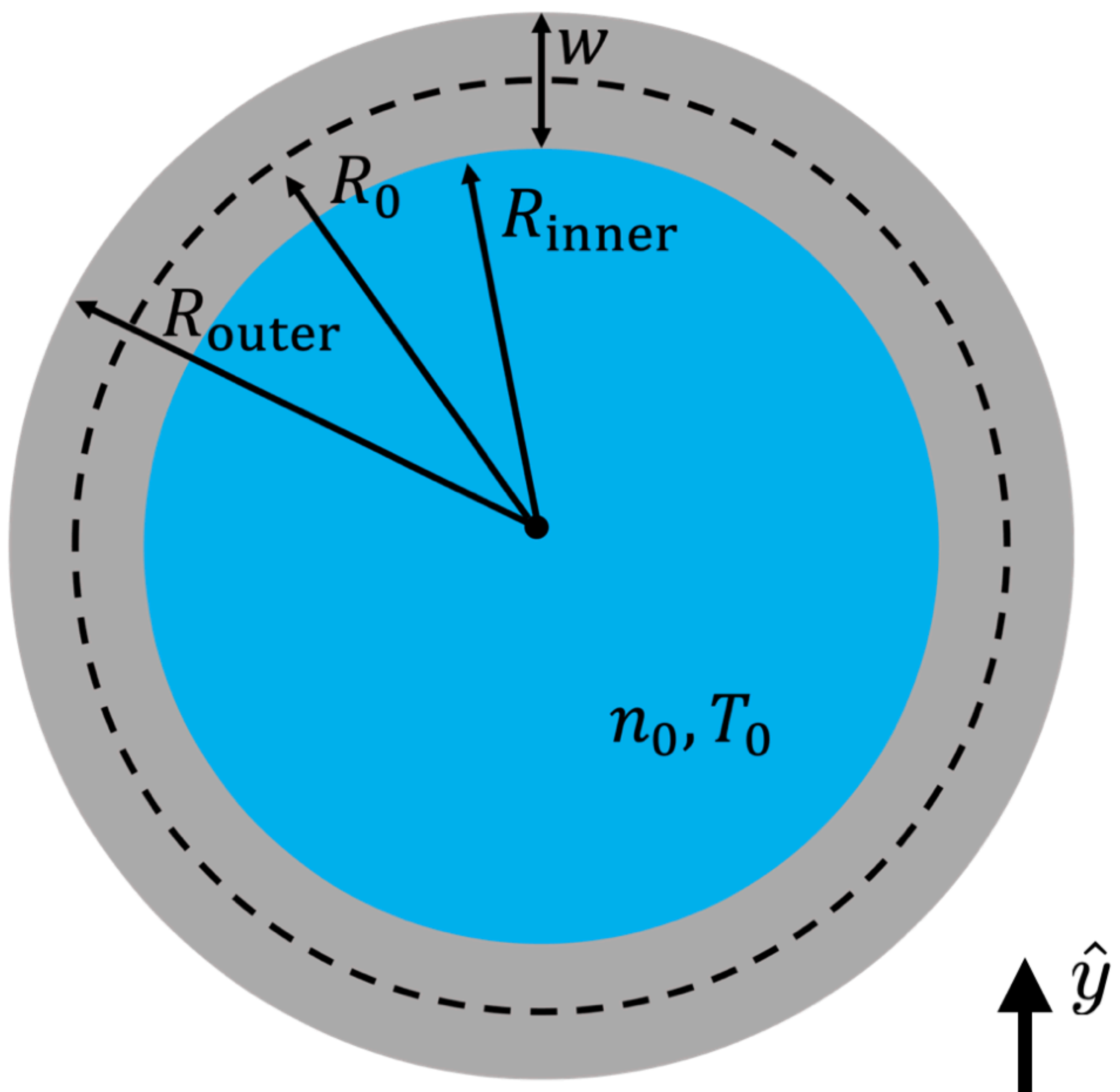


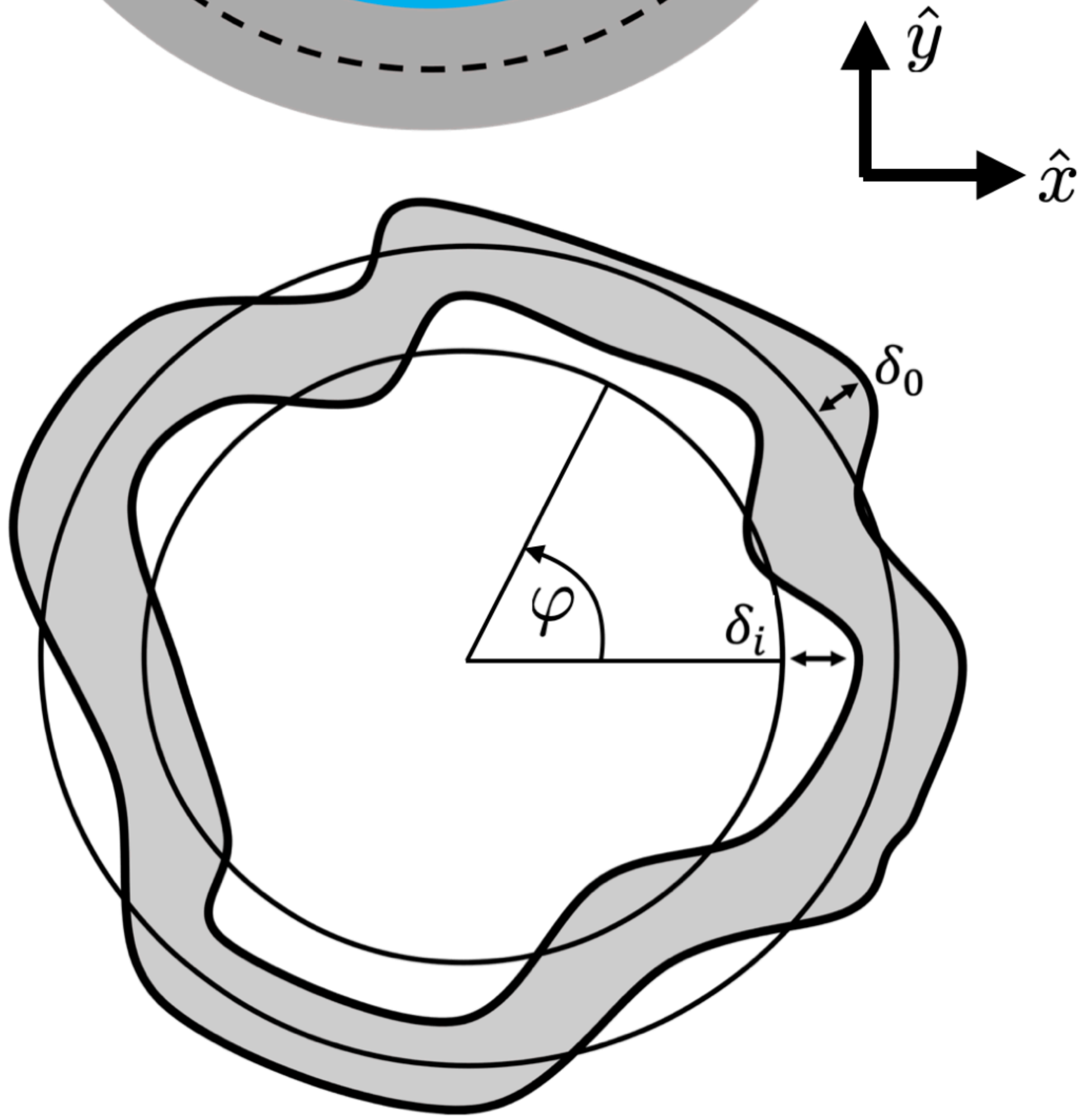

$$m_{\text{eff}} = 1/\omega_Q^2$$

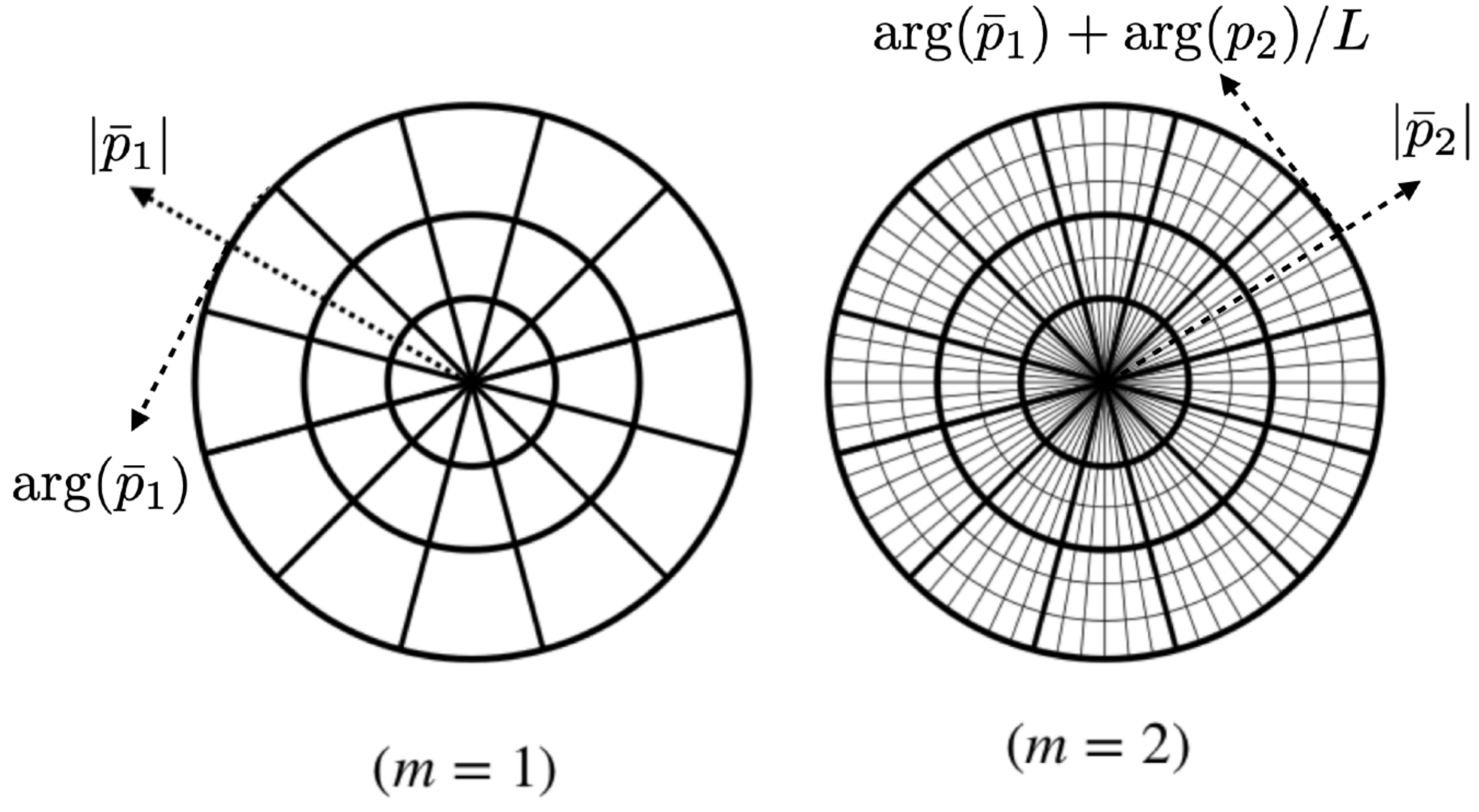
# Parameters of MHD model of liner implosions (MagLIF)






$T_0$ = implosion adiabat = compression ratio

$\mathrm{AR} = w/R_0$ = liner aspect ratio = acceleration

$\Delta$ = magnitude of liner perturbation, fraction of thickness $w$

$\varphi_{in}$ = initial phase angle of m=n liner perturbation

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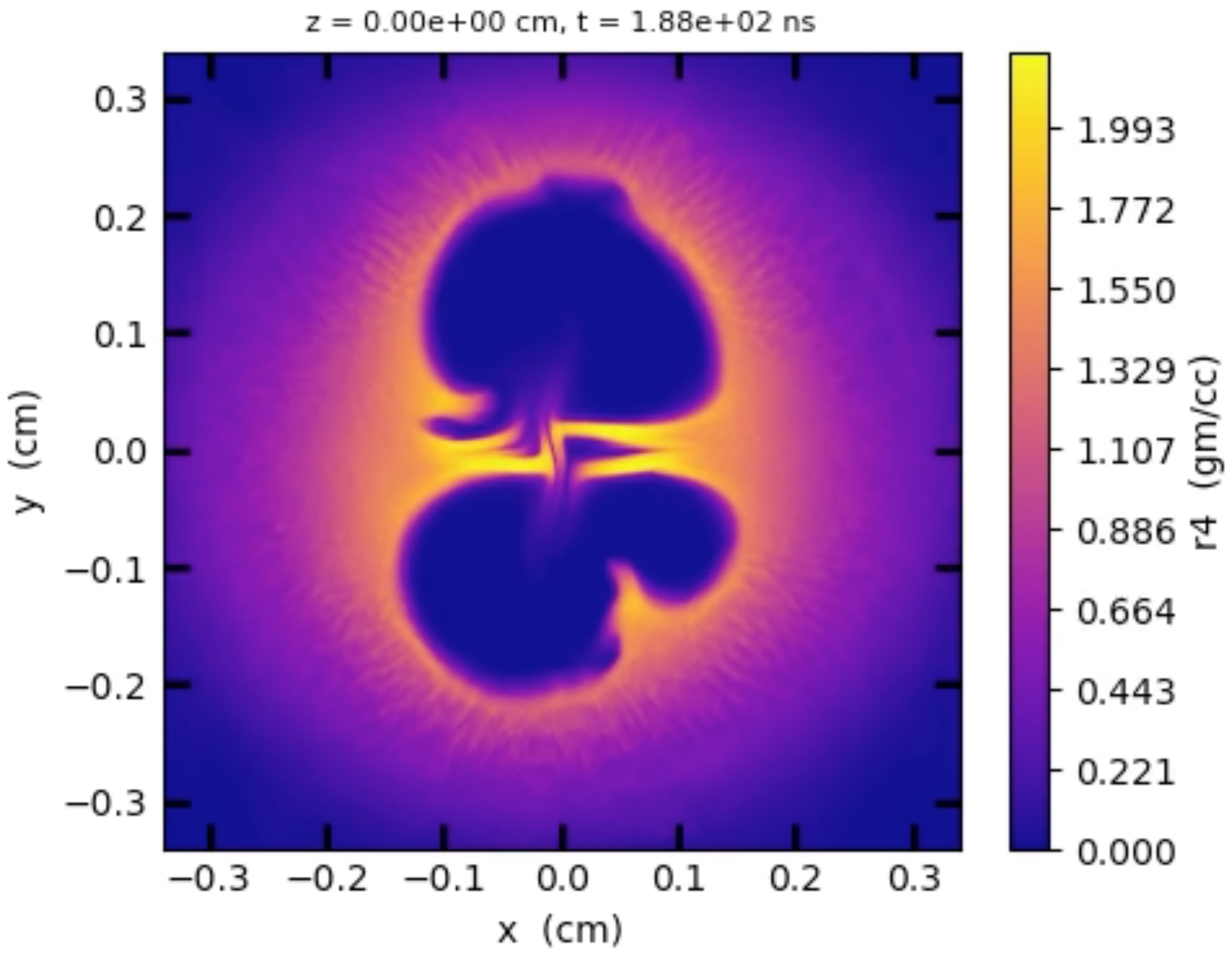
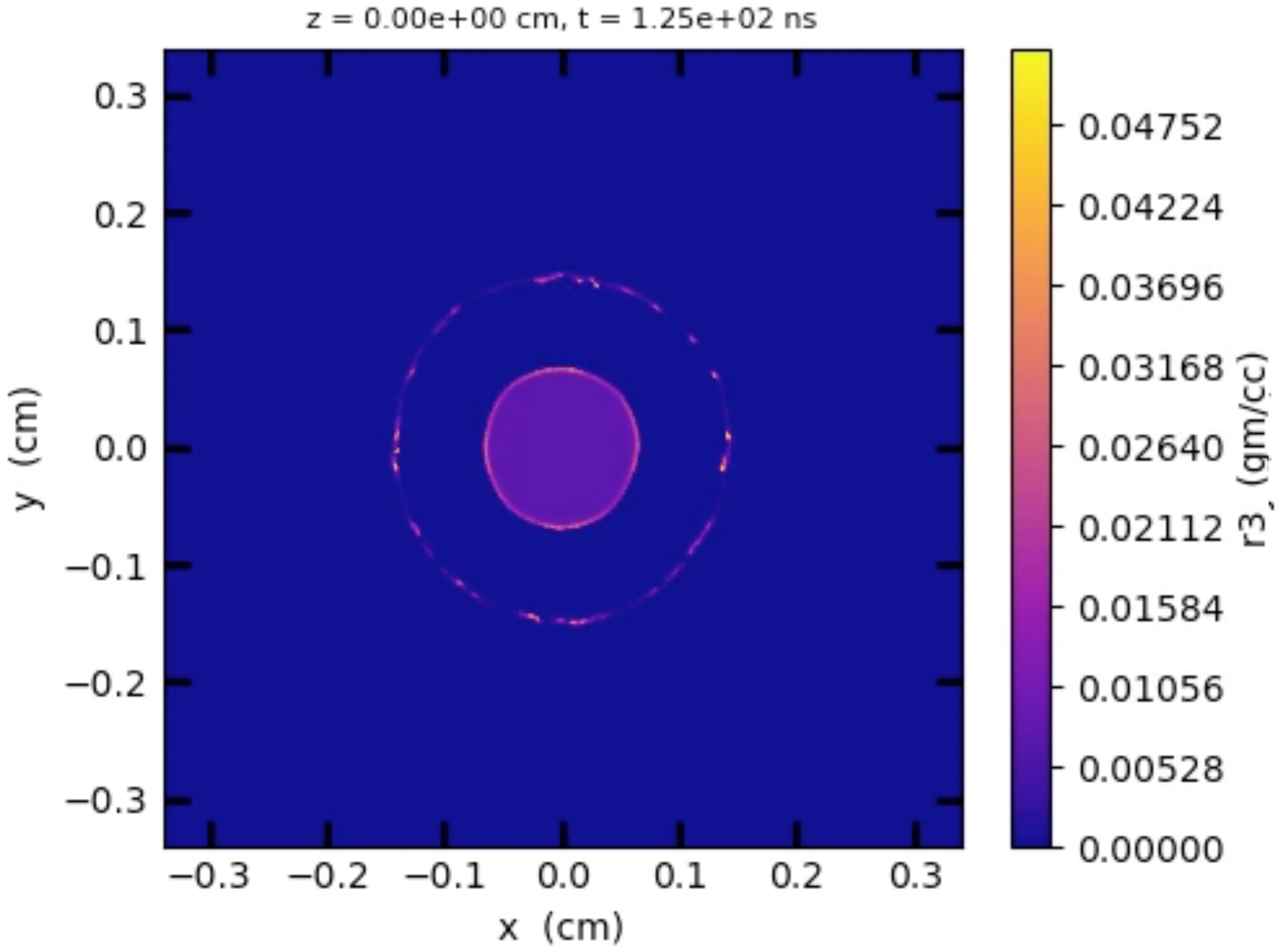
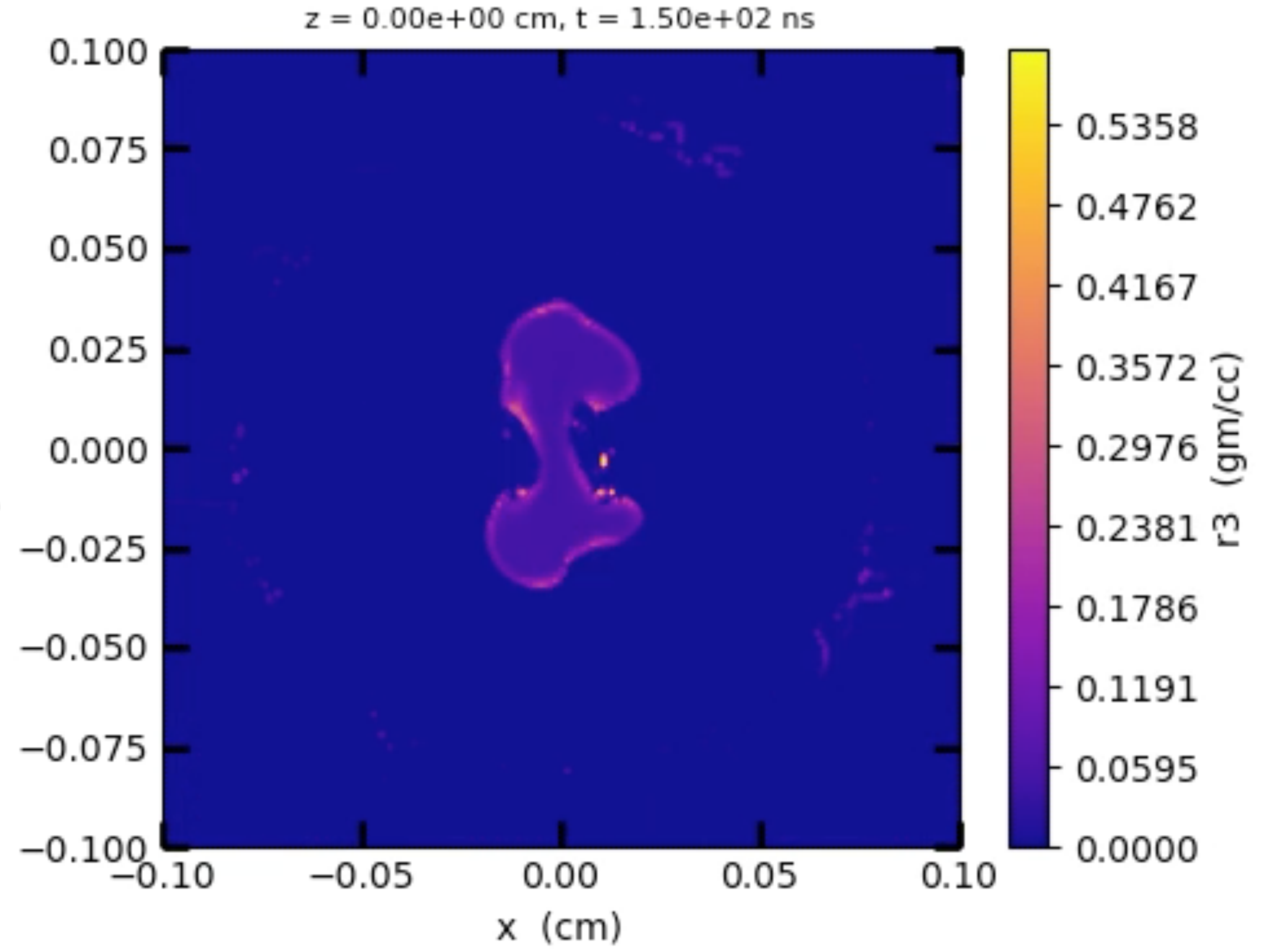
# Example MHD implosion evolution

$$\mathrm{AR} = 3, T_o = 610\ \mathrm{eV}, \Delta = 1\%$$




M.E. Glinsky and K. Maupin, Computational Mechanics **72**, 291 (2023)

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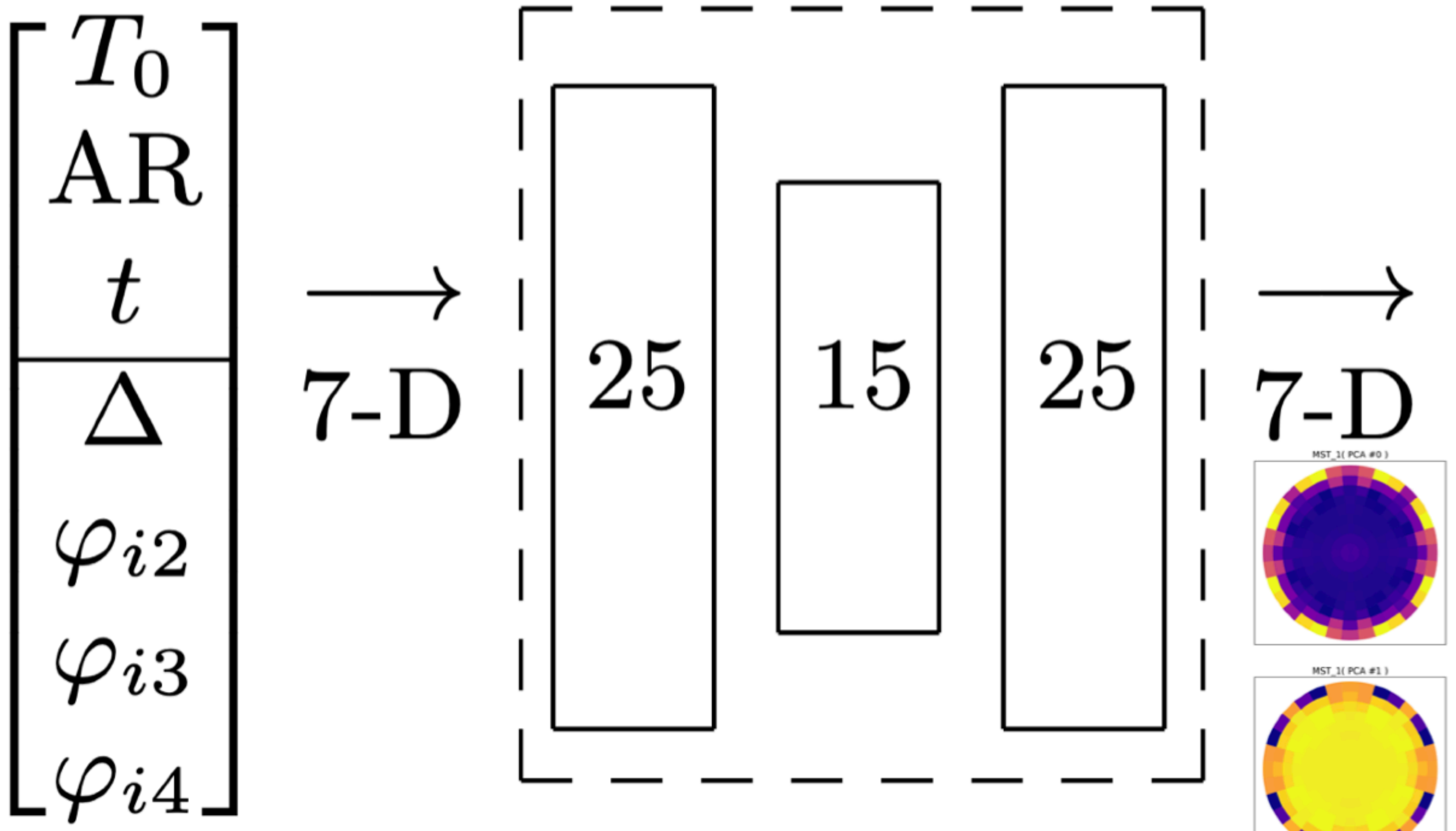
# MLDL architecture, that is log(MST(log))/PCA/MLP pipeline

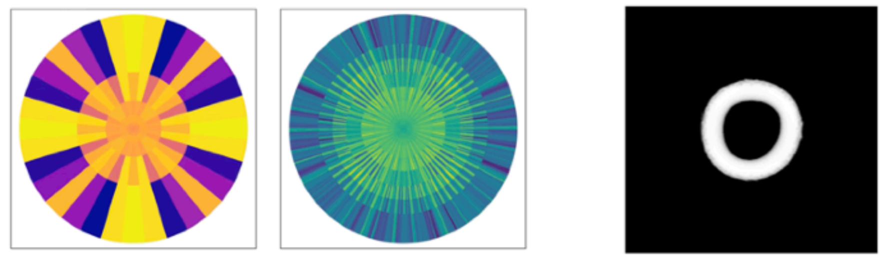
Z transform

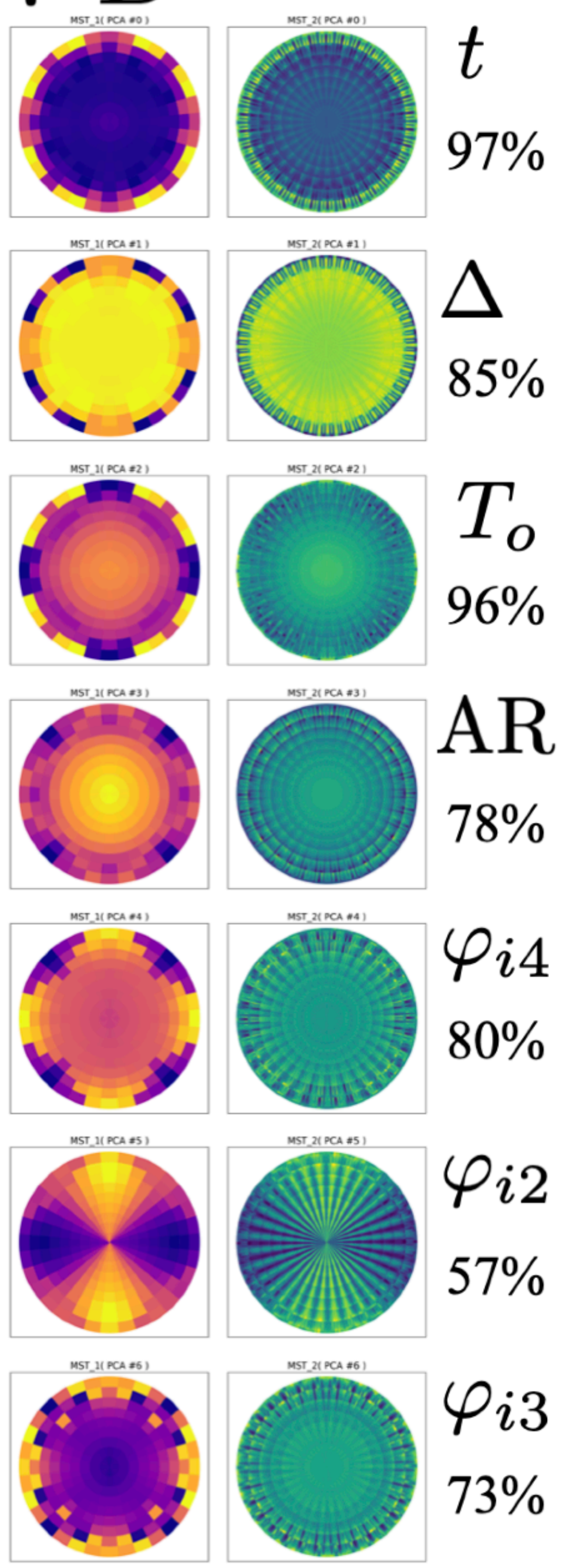
solution of Hamilton-Jacobi (Bellman) equation, or Laplace's equation, or geodesics

MLP/NN



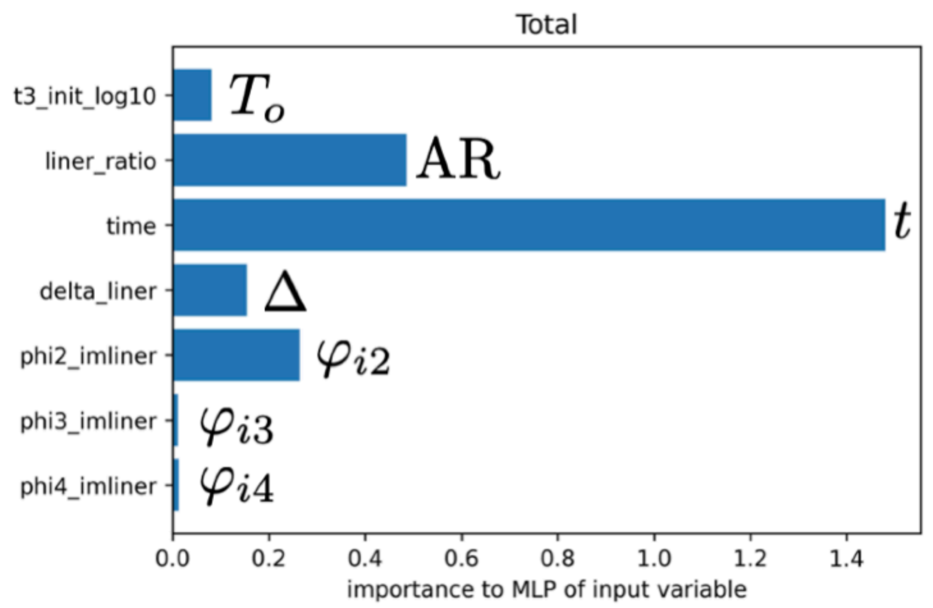

PCA | $\log_{10}$MST - mean | $\log_{10} n_\ell$



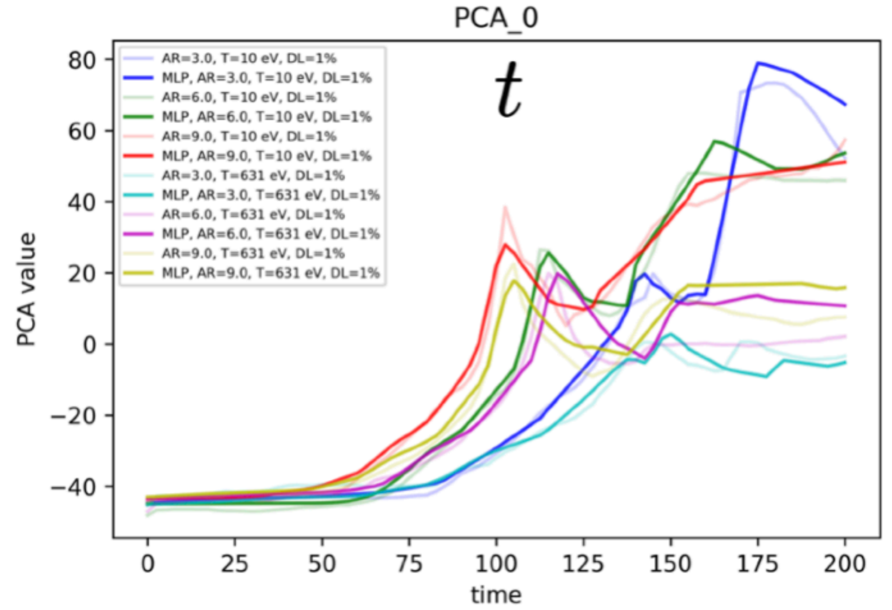



MLP/NN total score = 81%

PCA variance explained = 94%




permutation feature importance analysis




predicted vs. actual Principle Component time evolution

- **ensemble** of 539 simulations generating 87,318 256x256 images taking 200k core*hrs
- **training** took 16 GPU*hrs for MST, 27 GPU*hrs for WPH, 1 core*sec PCA, 20 core*sec MLP
- **surrogate** (0.1 core*sec) accelerates a simulation (360 core*hrs) by a factor of $10^7$

# Including phase with Wavelet Phase Harmonics (WPH) to generate correlated fields

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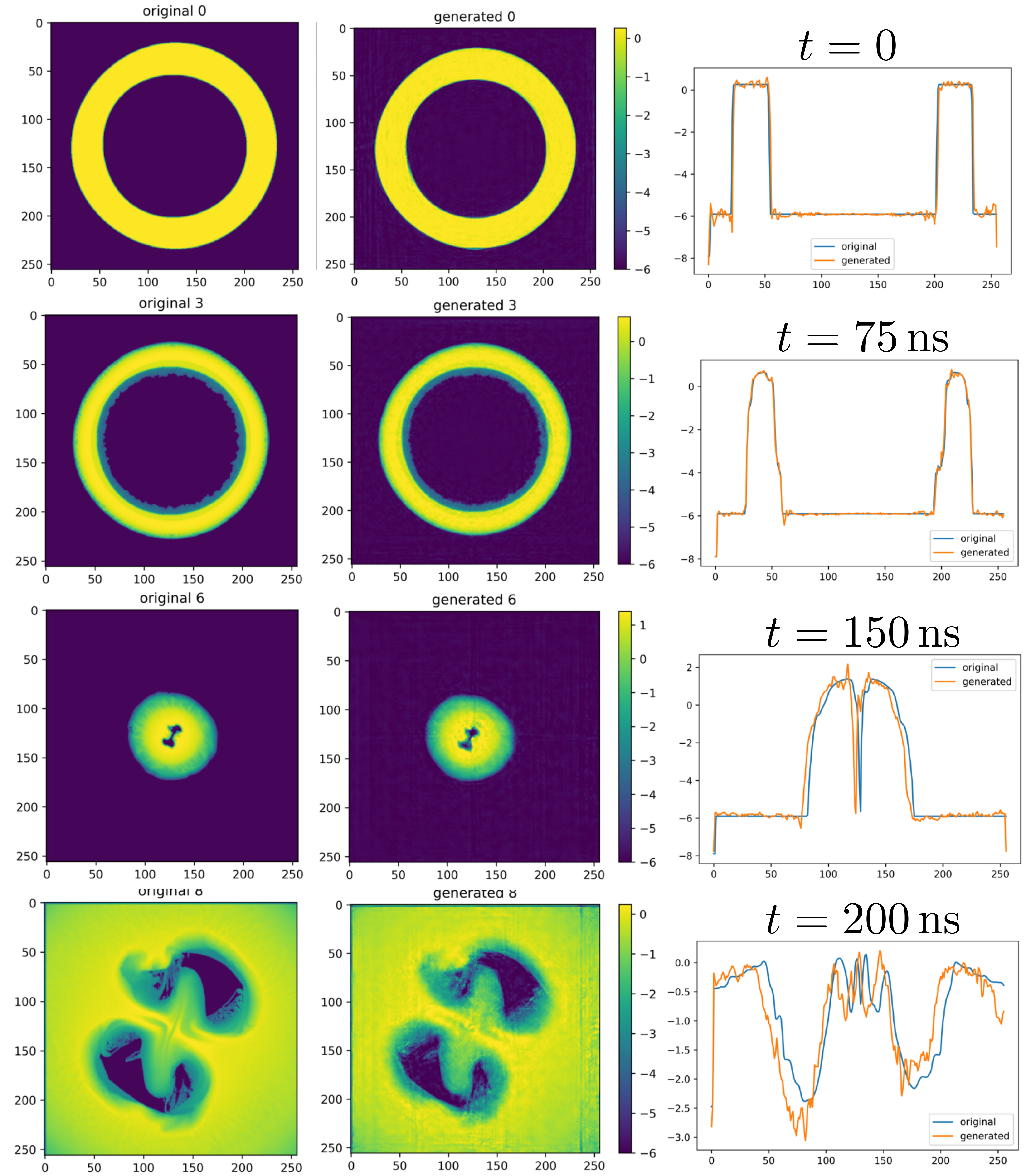
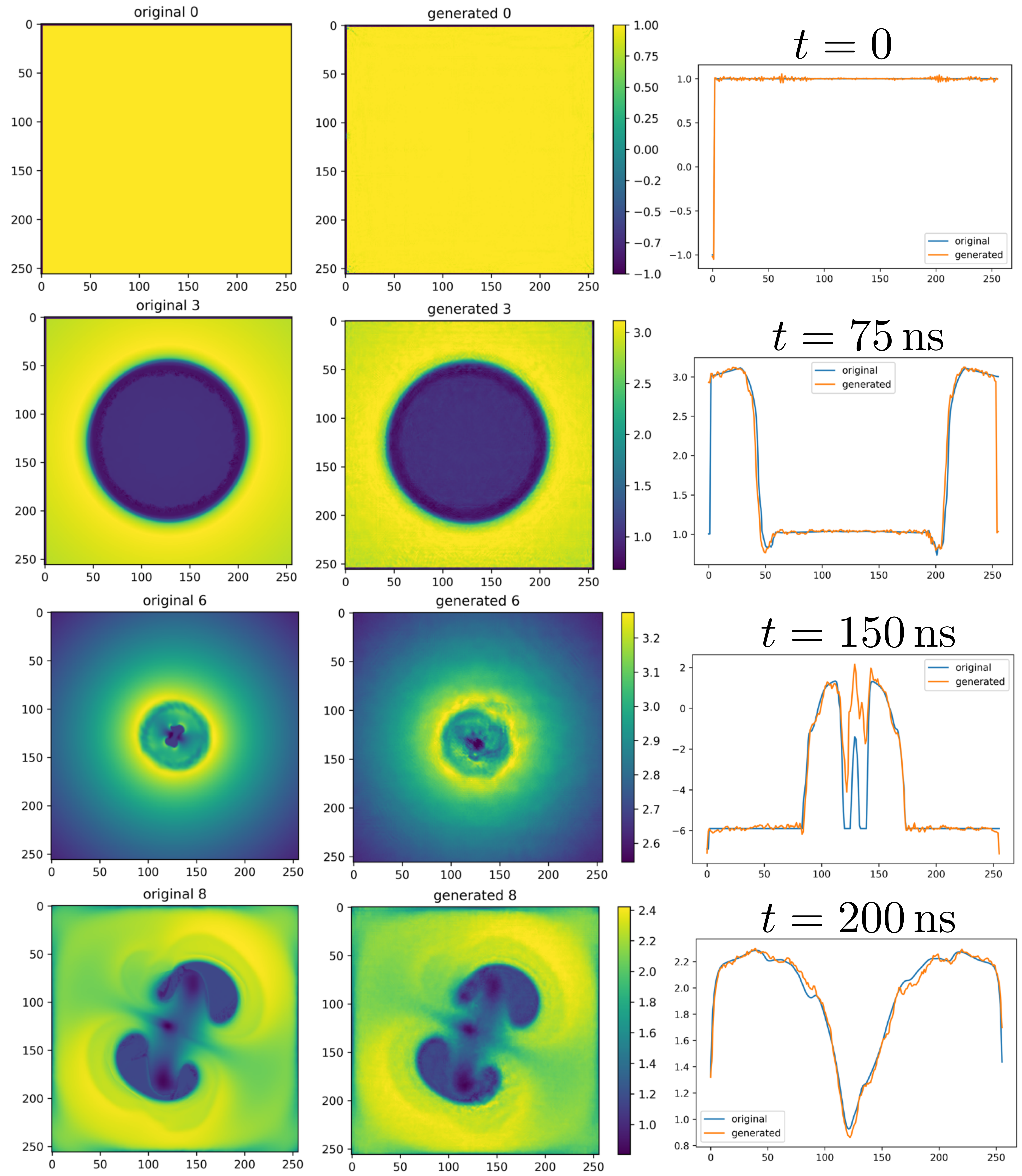
$n_l$




$B$



# Physical interpretation of results — inverse cascade into dipole (m=2) self organized state or emergent behavior

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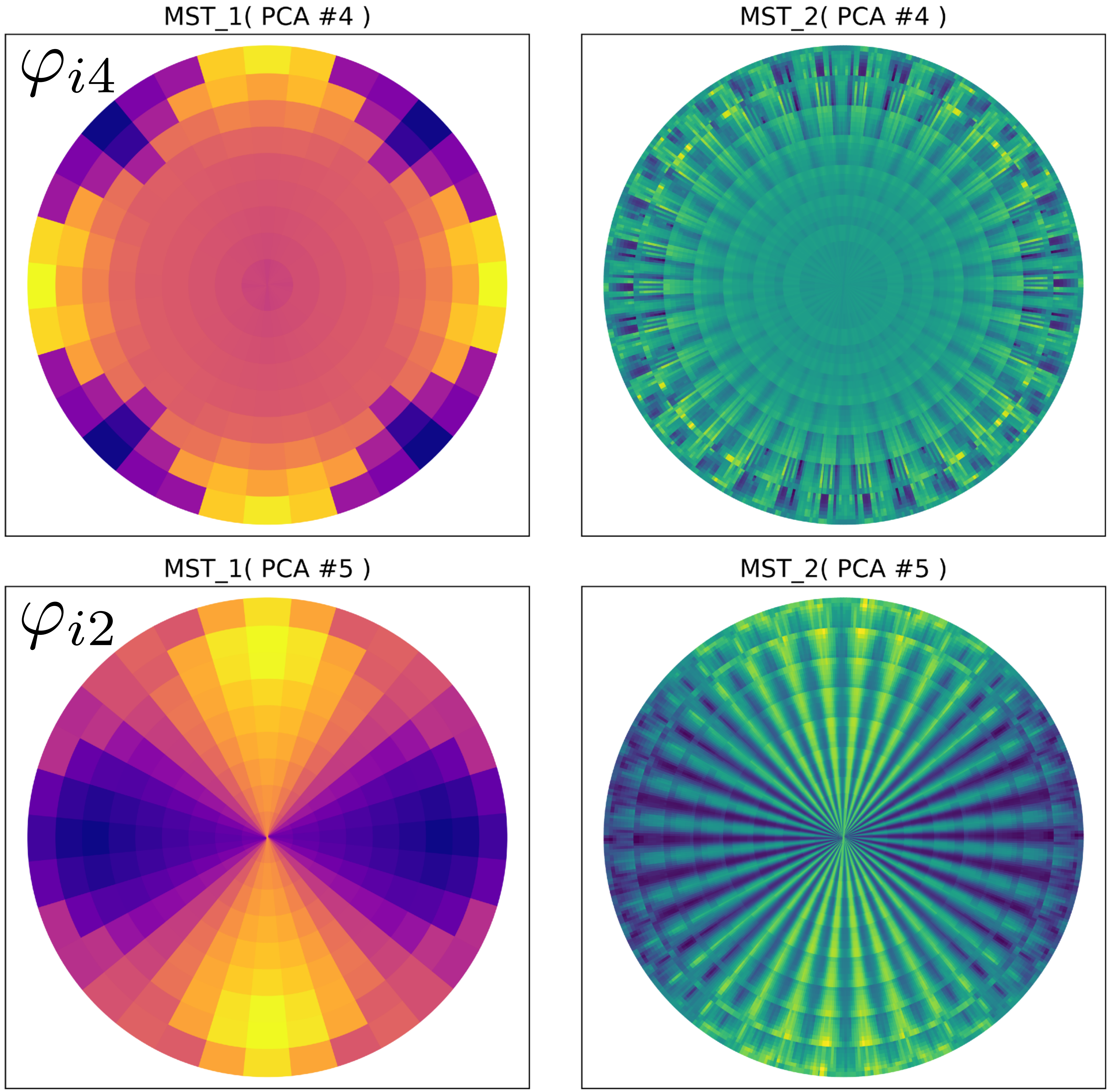


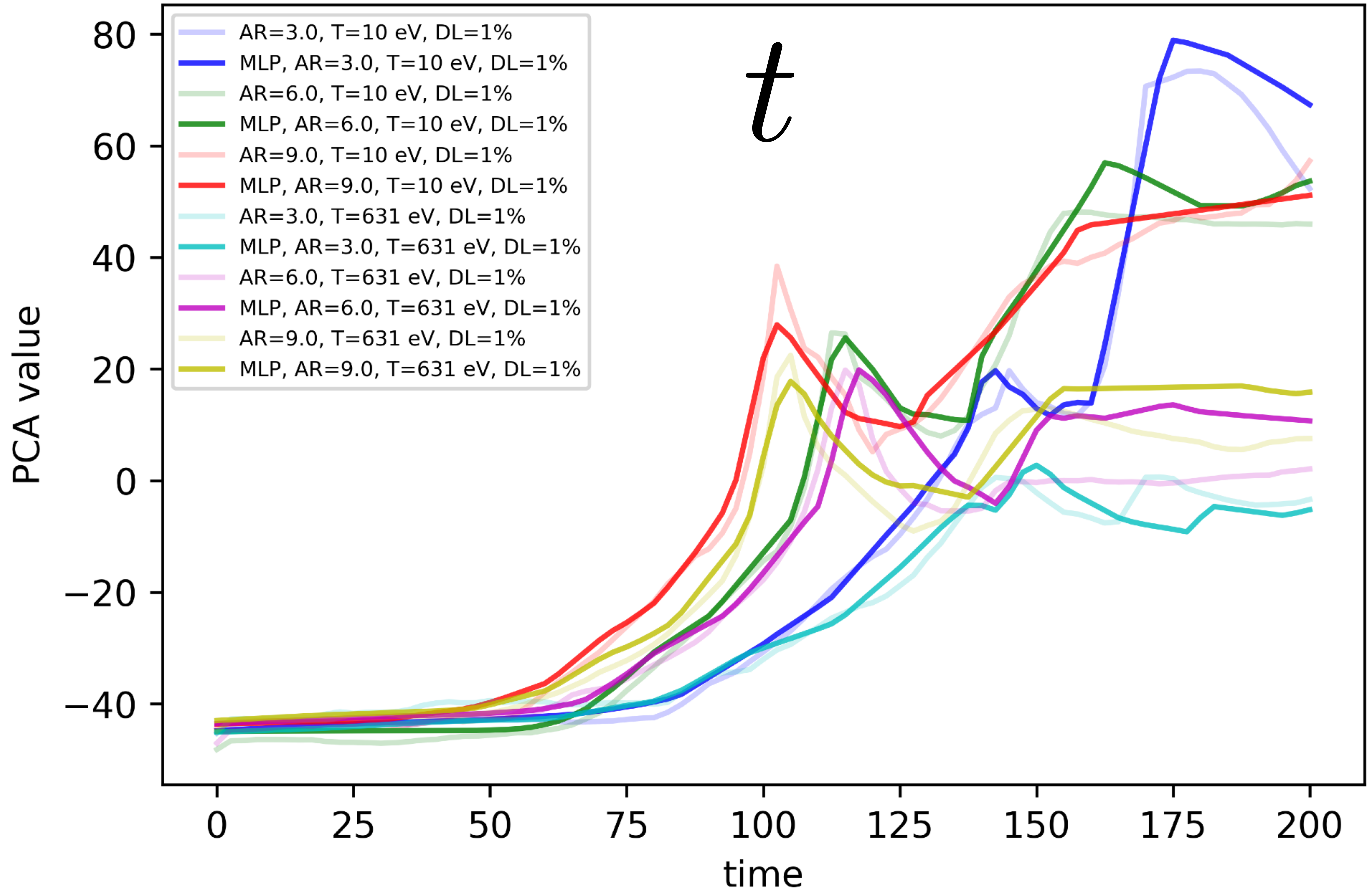


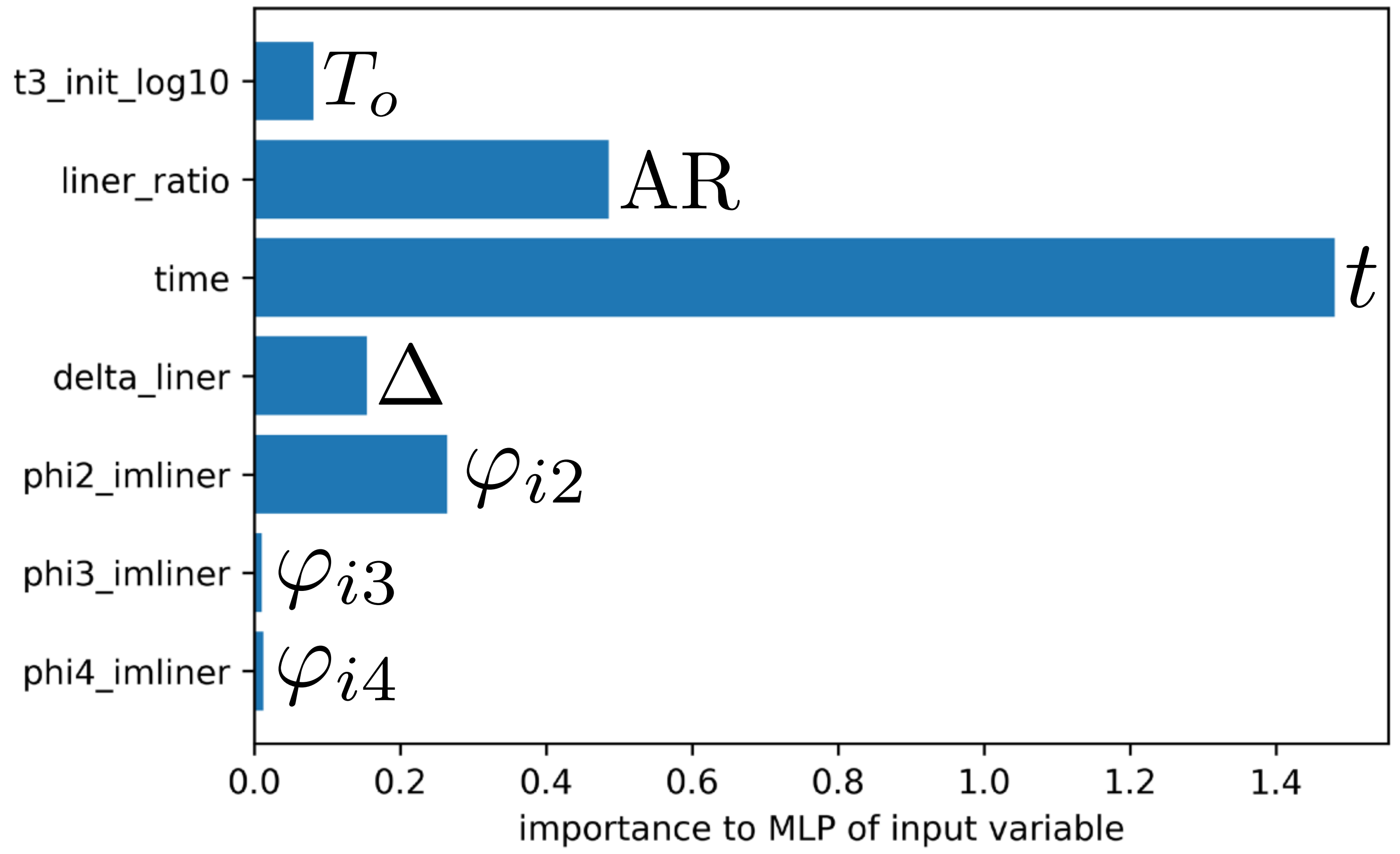

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# Conclusions — the topology of dynamical manifolds

- transformational method that is finding the time invariant coordinates (P,Q;E(P)) that are the solution to the Hamilton-Jacobi equation
  - ✦ Reduced Order Model (ROM) of Artificial Intelligence (AI)
  - ✦ what Generative Pre-educated Transformers (GPTs) are learning
  - ✦ what Deep Reinforcement Learning (DRL) is learning, based on Hamilton-Jacobi-Bellman Equation
  - ✦ what the HST(deep deconvolution)/PCA/MLP(decoder) computational pipeline is finding:
    - primally, The Golden Rule of collective behavior and philosophy — **understanding the global consequences of local actions** — knowing the path of least action
    - “the collective acts as one”, “a rising tide floats all ships”, “tous pour un, un pour tous”, “do unto others as you would have them do unto you”
    - the essence of well educated intelligence — the ability to think critically, that is strategically (example, play the beautiful Brazilian game of football)
      - contrasted to trained stupidity — reflexive action (example, maximize website hits without regard to the consequences)
    - The Inherent Goodness of Well Educated Intelligence
    - direct and computationally efficient
  - ✦ geodesics of the motion (minimum action paths)
  - ✦ analytic function H(β) that is solution to Laplace’s Equation, Lie group symmetries
  - ✦ topology of low dimensional dynamical manifold β*, that is **topological discovery**
- HST has physical meaning and interpretation, finds solution to Renormalization Group Equations (+PCA), extrapolates, supports causal analysis
- HST is a excellent metric, that exposes the:
  - ✦ dynamical manifold and its group symmetries
  - ✦ fundamental geometrical objects or relaxed states or topology, which are:
    - ground states of Quantum Field Theory (QFT)
    - attractive manifolds of nonlinear dynamics
    - emergent behaviors and self organizations of complex systems
    - Taylor relaxed states and BGK modes of plasma physics
    - poles and branch cuts of control theory and complex analysis
    - **homology classes of topology**

- **HST/PCA/MLP enables topological identification of fields in:**
  - ✦ **physics**
  - ✦ **complex systems and nonlinear dynamics**
  - ✦ **economics and finance**
  - ✦ **Artificial Intelligence (AI)**

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# BACKUP SLIDES

Further information

# Relationship to S-matrix and Wigner-Weyl transform

## Heisenberg's S-matrix

The concept of the S-matrix was first introduced by Werner Heisenberg in 1943 [8]. He lost interest in it, most likely, because he lacked concepts of analyticity. It was picked up again by Chew and Low [9] in 1955 and was most completely developed by Lev Landau [10] and Richard Cutkosky [11] by 1960. A good survey of this work was written by Chew [12]. This work was based on the concepts of analyticity and unitarity. It further used the expansion of an exponential generator employed by Richard Feynman in the path integral approach to quantum field theory. Furthermore, it evaluated the resulting integrals using a Fourier basis. It had no physical basis for the analyticity. Because of this, the theory was incomplete and did not form a well defined theoretical structure. It was also very difficult to calculate, and was not evaluated past second order. This line of research was abandoned in favor of the path integral formulation.

The work presented in this paper addresses these deficiencies. First, analyticity is the key to this work and has a deep physical basis coming from the isomorphism between Hamilton's equations and the Cauchy-Riemann equations. There is no need to enforce unitarity, since analyticity leads to unitarity. The exponential generator is replaced by a logarithmic generator so that the dynamics are constrained to a linear subspace. A wavelet basis with local support is used in place of the Fourier basis so that the integrals do not suffer from infinities, more simply said, they are compatible with the evaluation of integrals on manifolds. The analytic function $H(\beta)$ specifies the topology of the dynamical manifold.

## Wigner-Weyl transform

This work is also closely related to the Wigner-Weyl transformation [13–15]. They wanted to transform from the cotangent bundle to $\mathbb{R}^n$, then solve an ODE. The problems were they: (1) used a global Fourier basis, (2) only calculated to second order, and (3) did not include the logarithmic transform. This resulted in complicated corrections to the commutator, divergences in the evaluation, and an incomplete transformation.

The HST follows the basic philosophy of Eugene Wigner and Hermann Weyl of transforming from the cotangent bundle to $\mathbb{C}^n$, but uses an orthogonal local partition of unity based on coherent wavelet states to evaluate the integrals on the dynamic manifold, and embeds the complex logarithm in the transformation so that the dynamical manifold is aligned with $\mathbb{C}^n$, where $n$ is the number of fields. The beauty of the HST is that the ODE is Laplaces's equation (the Cauchy-Riemann equations) and the dynamical motion is simply geodesic motion, like for general relativity, given the topology of the dynamical manifold. Dynamics and Quantum Field Theory has been reduced to a matter of geometry — the geometry of physics [16].

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# Multi Layer Perceptron (MLP) regression

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# Support Vector Regression (SVR)