Interdepartmental letterhead
Mail Station L- 43
Ext: 2-3803

September 19, 1985
RP-85-104

M E M O R A N D U M

To: Distribution

From: M. Glinsky MEG

Subject: Optical Resolution of Curved Mica Crystals

Attached is a report on the measured optical resolution of a curved mica crystal and the effect of the quality of crystal curvature on the x-ray resolution of curved crystal spectrometers.

MG:lr

Distribution:

L-Division Group Leaders
G. Chandler
X-ray Measurements Group
R-Project Group

University of California
Lawrence Livermore
National Laboratory

Optical Resolving Power of a Curved Mica Crystal

by
M. E. Glinsky

ABSTRACT

The quality of the curvature of a 100 μm thick mica crystal is measured. The mica crystal is cylindrically curved to a 5.08 cm radius of curvature by clamping it between two machined forms. The average angular deviation from what would be expected of a cylindrically curved crystal over a distance of 510 μm is measured to be less than 2.1×10^{-3} radians.

INTRODUCTION

When curved crystals are used in spectrometers the quality of the curvature can degrade the resolution by smearing the expected dispersion of the X-rays. The purpose of this investigation is to measure the quality of the curvature attained in curving a mica crystal by clamping it between two machined dies, and to relate this error in curvature to the limitation it puts on the resolving power of the spectrometer.

The quantity used in this investigation to specify the amount of imperfection in curvature is σ_θ. Given a desired angle of the surface, θ_C, and the actual angle of a surface, θ_t, at an angle θ (figure 1a), σ_θ is defined in the following way:

$$\sigma_\theta^2 = \int (\theta_t - \theta_C)^2 \, d\theta \; / \int d\theta \qquad (1)$$

To measure the imperfection in curvature, a bar resolution pattern reflected off the crystal is photographed. If the crystal can only resolve bars separated by a distance, x, or greater, the imperfection in curvature is given by

$$\sigma_\theta = R_\beta x \; / \; R_\alpha f \qquad (2)$$

where R_β is the distance from the focal spot to the film, R_α is the distance from the curved crystal to the film, and f is the distance from the crystal to its focal spot. The geometry governing this equation is shown in figure 1b. Assume that two rays from neighboring resolution bars approach the crystal parallel to one another. The two rays strike the crystal at points A and B respectively. After being reflected from the crystal they strike the film at points A' and B'. When the two rays can no longer be resolved a ray from point A is reflected to B' because of the imperfection in curvature. The condition for this to happen is expressed in equation 2.

The effect of σ_θ on the resolution of a curved crystal X-ray spectrometer varies as a function of the distance of the film from the focal point of the crystal, R_β, in the following way:

$$\omega = \sigma_\theta f / R_\beta \qquad (3)$$

where ω is the smallest angular deviation which can be resolved (figure 1c). The governing law of this relationship is that X-rays will always be reflected at their bragg angle. The area of the crystal which will reflect the X-rays is broadened by σ_θ. As R_β increases, the width of the beam stays constant decreasing its angular width allowing smaller angles to be resolved. In construction of a spectrometer the value of σ_θ should be small enough so that ω is less than the width of the crystals rocking curve.

The condition for image formation from reflection from a circle off axis is

$$1/S + 1/S' = 1/f = 2/R\cos\theta \qquad (4)$$

The variables are defined in figure 2a. It is obvious that the focal points lie on a circle of radius R/4 tangent to the reflecting surface (figure 2b). If the grid is illuminated by a coherent source at infinity, the general Abbe theory {1} predicts that an image of the grid will be formed at the grids conjugate plane. The magnification of the grid will be

$$m = -S/S' \qquad (5)$$

where the grid is placed at S and the image formed at S'. Assume that the grid is placed near the focal circle (i.e., $S= f+\varepsilon$). This implies

$$S' \approx S/\varepsilon \qquad (6)$$

and

$$m \approx -S'/f \qquad (7)$$

Therefore, in the limit of $S' \gg f$, $S \approx f$ and $m \approx -S'/f$. These imaging properties of the curved crystal allow a coherent laser beam to be used to illuminate a resolution chart a focal length away from the crystal. The image of the grid will be formed at $S' > \sim 10f$ with magnification of the grid equal to $-S'/f$.

The area of the crystal which will influence the image of one of the resolution bars is roughly the width of the peak of the Fraunhofer diffraction pattern of the slit. The width of this peak, W_f, is given by

$$W_f = 2S\lambda/x \qquad (8)$$

where λ is the wavelength of the light and x is defined as in equation 2. The width of the slit is assumed to be x/2. This causes the angular deviations of the crystal surface to be averaged over a distance, W_f.

EXPERIMENTAL

A laser beam from a He-Ne gas laser (wavelength 6328Å) widened with a beam expander is used to illuminate a resolution grid. The grid consists of four sets of lines 200 μm tall (figure 3a). Each set of lines extends for 1mm and is separated from the next set by 1 mm. The sets of lines have 5, 10, 15 and 20 line pairs per mm. The grid is placed a focal length from the crystal so that the image of the grid will be formed near infinity, greatly reducing the diffraction effects of the coherent light. The crystal is a 100 μm thick mica crystal 4 cm by 3 cm. The crystal is bent into a cylinder with a 5.08 cm radius of curvature by clamping it between two metal forms machined to the proper radius. The identification number of the crystal measured is R-SPM-004-3/4.

The pattern incident on the crystal is measured using the geometry of figure 3b. The image is recorded using Polaroid type 55 film then digitized by a micro densitometer. The vertical average of the image density is taken and plotted as a function of lateral position.

The image of the grid reflected by the crystal is then photographed using the geometry of figure 3c and processed like that of the incident pattern.

RESULTS

The pattern incident on the crystal with D_C = 2 cm, is shown in figure 4. The sets of lines have been widened as would be expected by Fraunhofer diffraction of one of the slits.

line pairs / mm	W_f (calculated)	Wf (measured)
5	127 μm	160 μm
10	253	250
15	380	370
20	506	490

The image of the grid reflected off the crystal was then recorded with D_C = 2 cm, θ = 35° and D_f = 10 cm. The results are shown in figure 5. 5 and 10 lp/mm are clearly resolved but the enlargement of 15 and 20 lp/mm in figures 6 and 7 show degradation of the patterns due to the diffraction not being well focused. To correct this problem, the crystal was translated sideways so that the 20 lp/mm section went through the center line of the crystal. The crystal was rotated to θ = 30° and the film moved to D_f = 24 cm to enlarge the image. The result is shown is figure 8. 20 lp/mm are clearly resolvable.

CONCLUSIONS

Greater than 20 lp/mm can be resolved by the curved mica crystal studied. This translates to $\sigma_\theta < 2.1\times10^{-3}$ radians. The X-ray resolving power of the crystal will not be limited by σ_θ until the rocking curve width, ω_C, is

$$\omega_C < 3.8 \times 10^{-3} \text{ radian*cm} / R_\beta$$

This assumes θ=45°. The measured value for ω_C {2} is 7.7×10^{-4} radians. This requires that R_β to be greater than 4.9 cm so that the resolution will not be limited by crystal curvature.

REFERENCES

1. Klein, M V.: Optics, Wiley New York, 1970, pp. 431-435.

2. Glinsky, M. E., Waide, P. A.: Resolving Power of Muscovite Mica (002), RP-85-91, 1985.

Figure 1a.

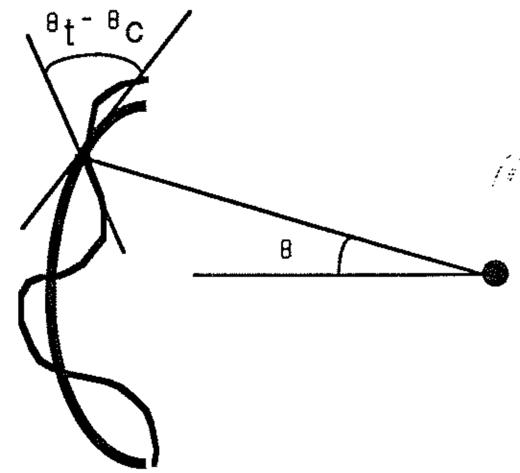
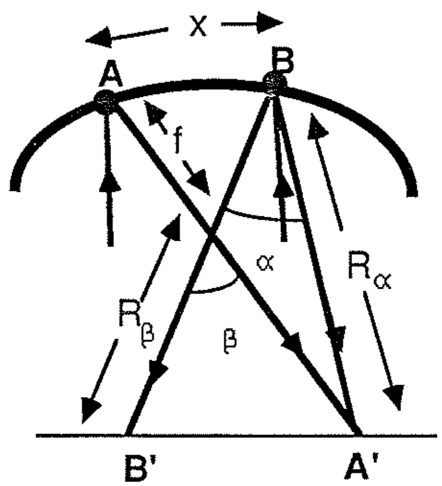


Figure 1b.

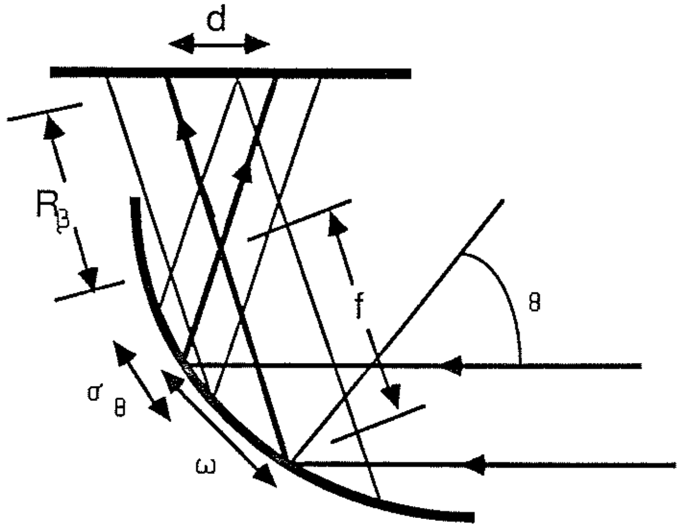


$$\alpha = 2\sigma_\theta$$

$$\beta = \frac{x}{f}$$

$$\beta R_\beta = \alpha R_\alpha$$

Figure 1c.



R = radius of curvature of crystal

$$\frac{R\omega \cos\theta}{f} = \frac{d}{R_\beta}$$

$$d = R\sigma_\theta \cos\theta$$

$$\frac{R\omega R_\beta \cos\theta}{f} = R\sigma_\theta \cos\theta$$

Figure 2a.

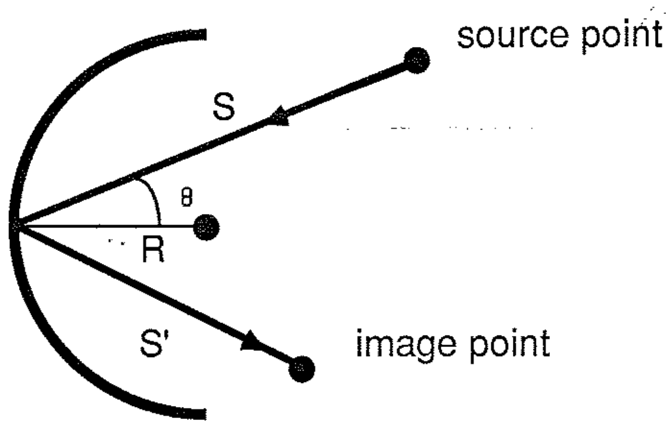


Figure 2b.

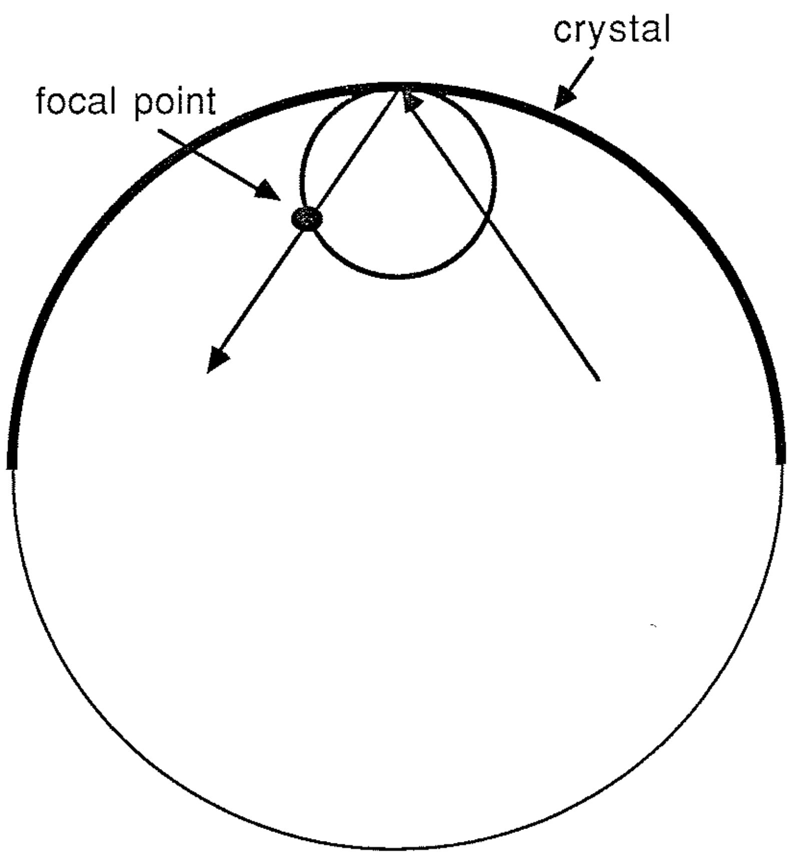


Figure 3a.

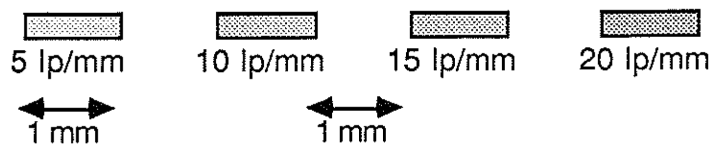


Figure 3b.

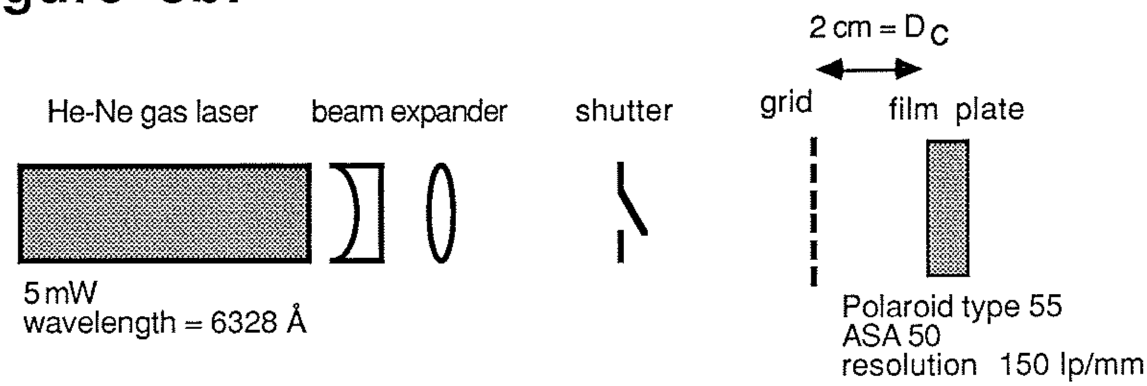


Figure 3c.

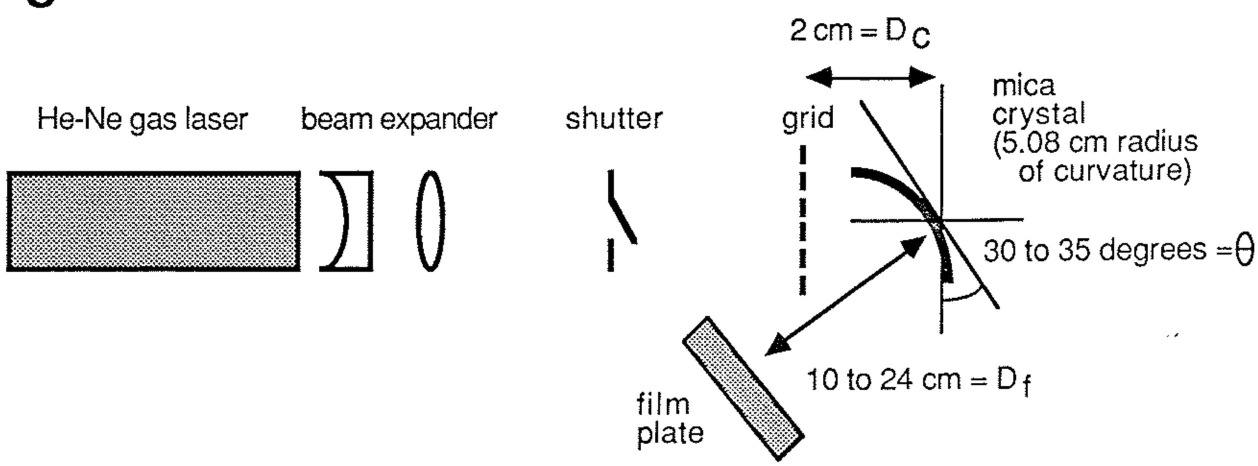
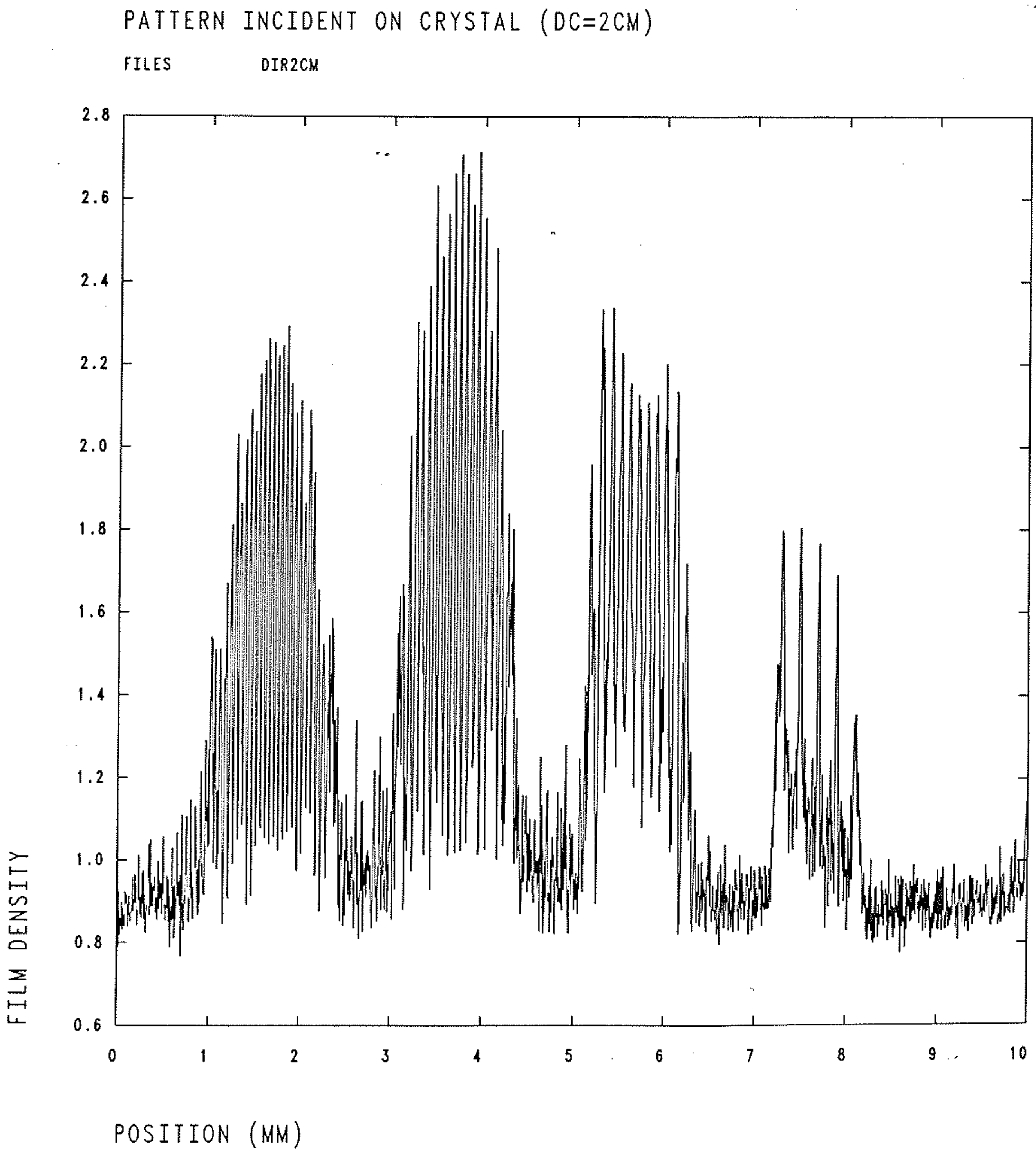
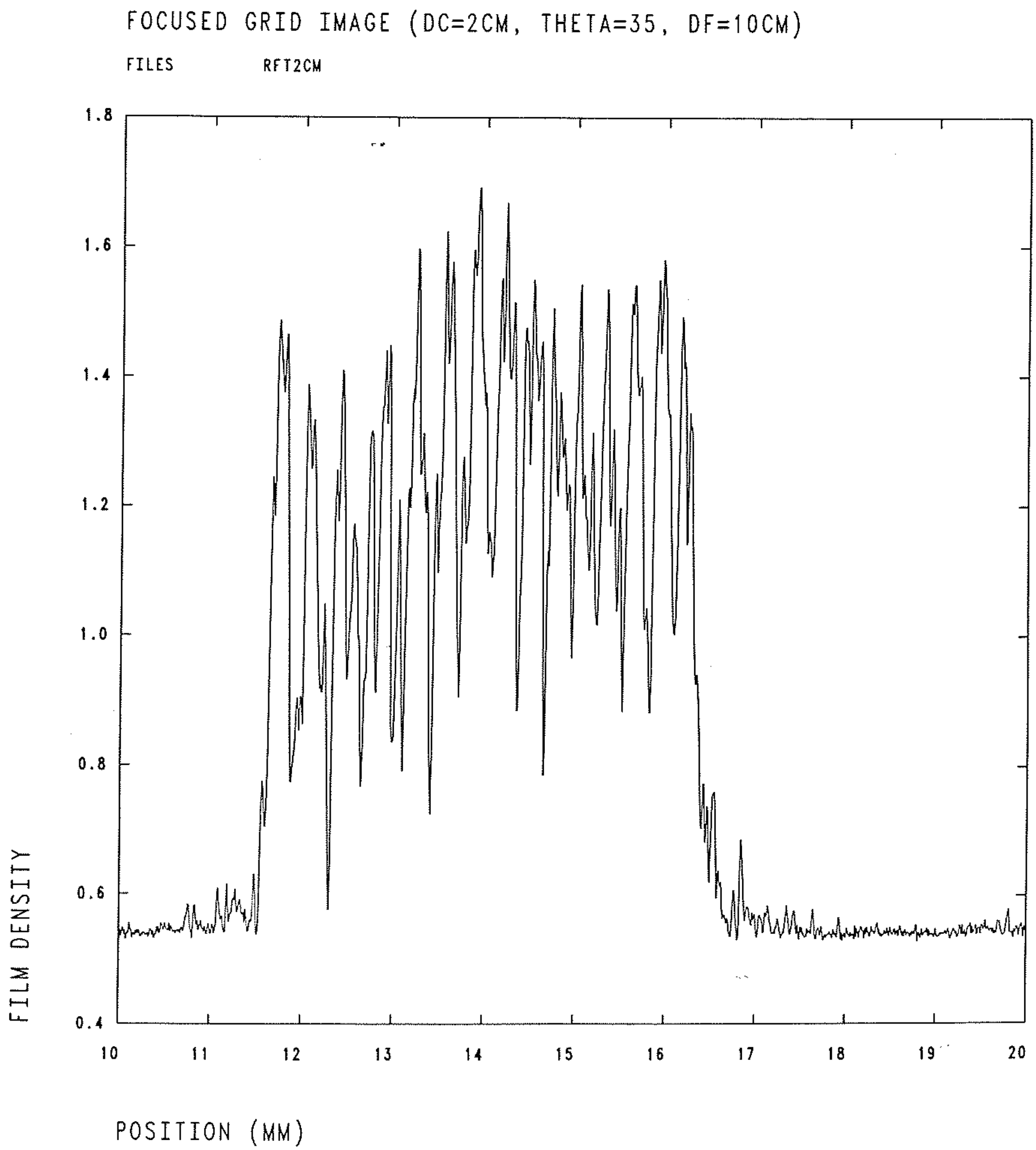


Figure 4.



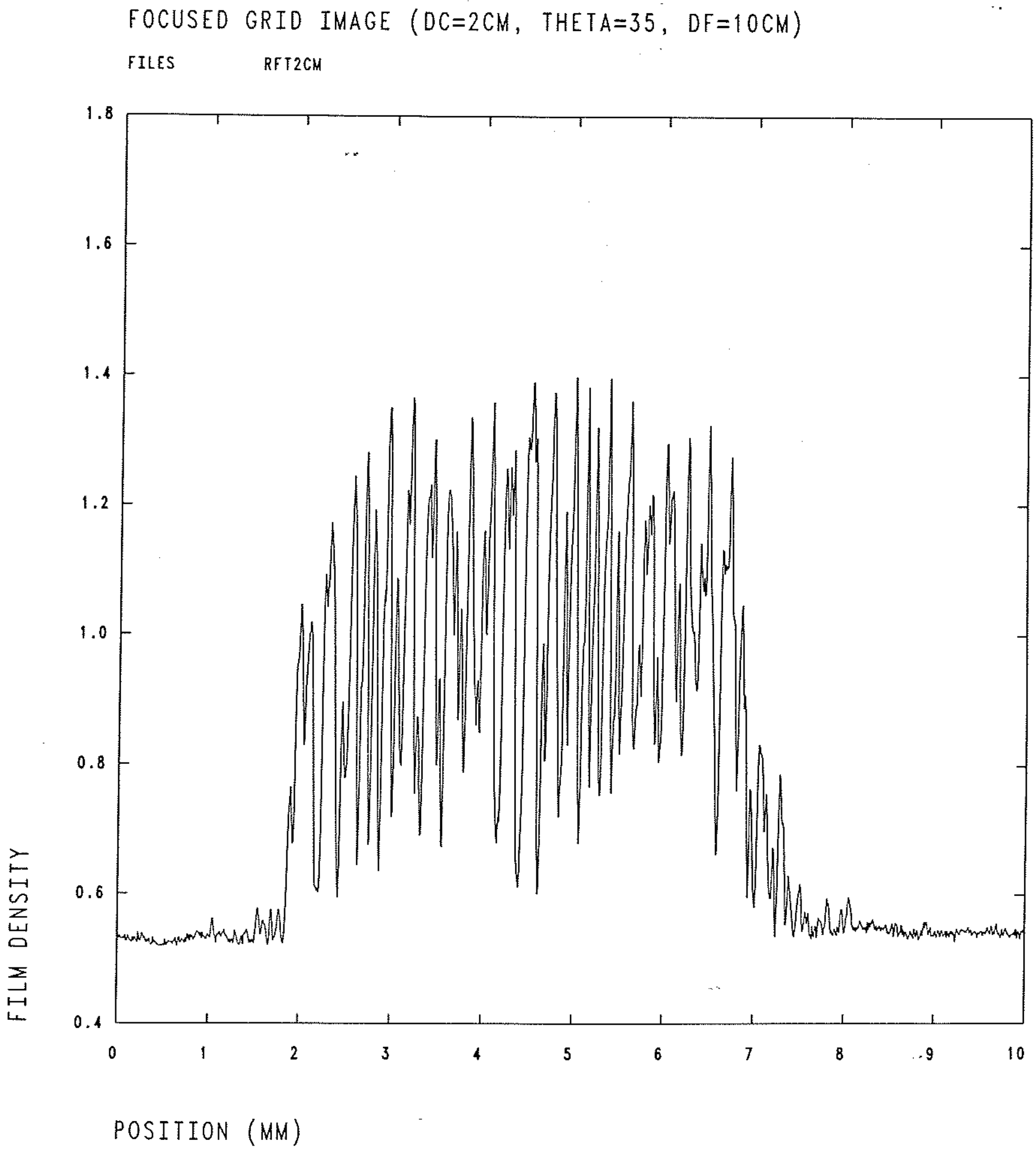
PATTERN INCIDENT ON CRYSTAL (DC=2CM)
FILES DIR2CM
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
FILM DENSITY
0
1
2
3
4
5
6
7
8
9
10
POSITION (MM)

Figure 6.



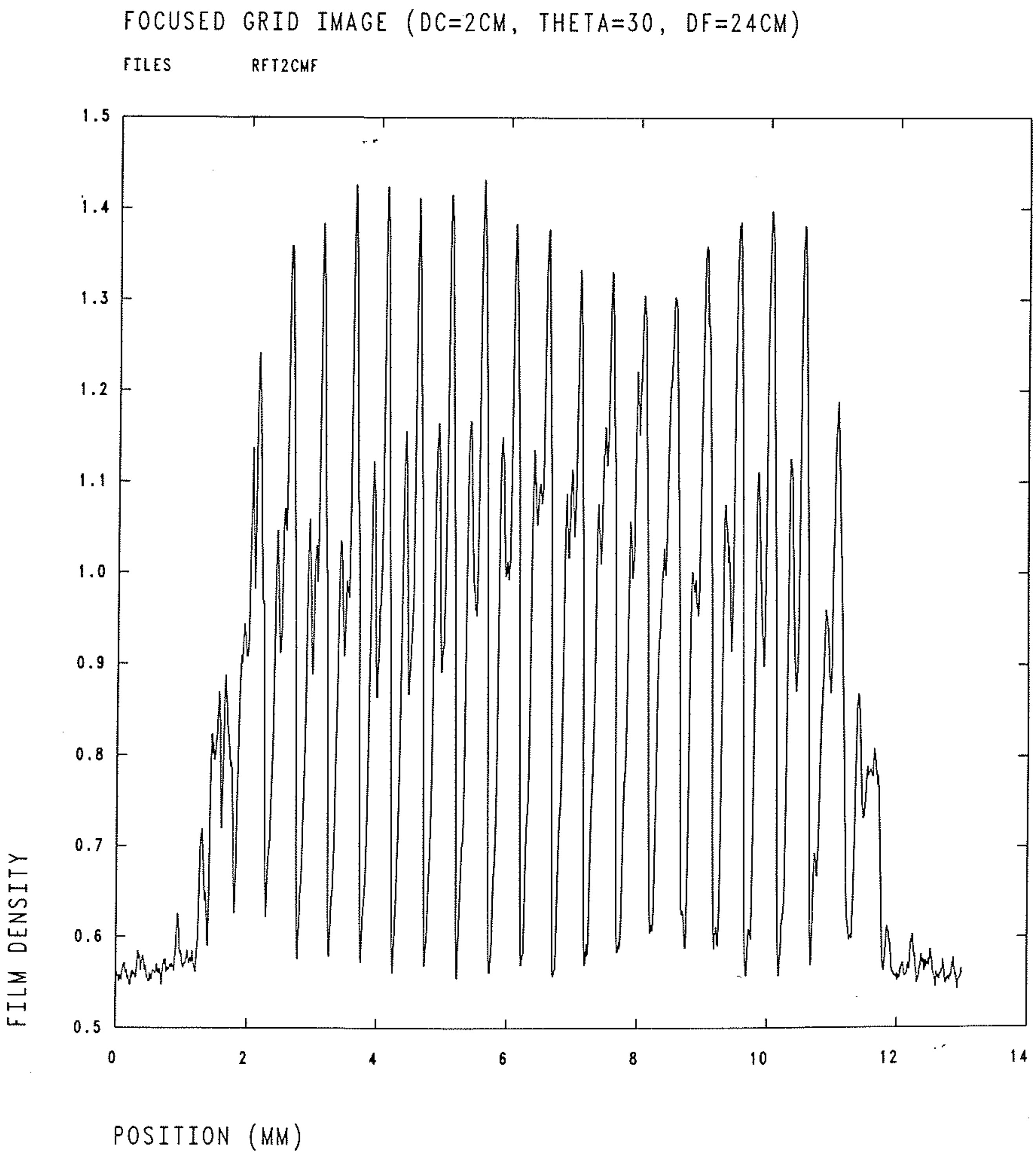
FOCUSED GRID IMAGE (DC=2CM, THETA=35, DF=10CM)
FILES
RFT2CM
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
10
11
12
13
14
15
16
17
18
19
20
FILM DENSITY
POSITION (MM)

Figure 7.



FOCUSED GRID IMAGE (DC=2CM, THETA=35, DF=10CM)
FILES RFT2CM
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
FILM DENSITY
0
1
2
3
4
5
6
7
8
9
10
POSITION (MM)

Figure 8.



FOCUSED GRID IMAGE (DC=2CM, THETA=30, DF=24CM)
FILES RFT2CMF
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
FILM DENSITY
0
2
4
6
8
10
12
14
POSITION (MM)